Booklet No. 85112446

JECA—2018

Subject : MATHEMATICS

Time : 2 Hours | *Full Marks* : 100

Instructions

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks *1*. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the OMR. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.
7. The OMRs will be processed by electronic means. Hence it is liable to become invalid if there is any mistake in the question booklet number or roll number entered or if there is any mistake in filling corresponding bubbles. Also it may become invalid if there is any discrepancy in the name of the candidate, name of the examination centre or signature of the candidate vis-à-vis what is given in the candidate's admit card. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, docu-pen, log table, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough Work must be done on the question paper itself. Additional blank pages are given in the question paper for Rough Work.
10. Hand over the OMR to the invigilator before leaving the Examination Hall.

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SPACE FOR ROUGH WORK

1. Let us apply translation so that the origin changes to the point (1, 1) and then turn the new axes through an acute angle $\theta = \tan^{-1}\frac{5}{12}$. The coordinate of a point P which remains invariant under the transformation is

(A) (−2, 3) (B) (2, 3) (C) (3, 2) (D) (− 3, 2)

2. The distance between the parallel lines represented by $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ is

(A) $\frac{4}{\sqrt{10}}$ unit (B) $\frac{6}{\sqrt{10}}$ unit (C) $\frac{3}{\sqrt{10}}$ unit (D) $\frac{2}{\sqrt{10}}$ unit

3. Equation of a line which is perpendicular to the line common to the two pairs of lines given by $3x^2 + xy - 4y^2 = 0$ and $3x^2 + 7xy + 4y^2 = 0$ is

(A) $4x + 3y = 0$ (B) $3x - 4y = 0$ (C) $4x - 3y = 0$ (D) $3x + 4y = 0$

4. A line joining two points $A(1, 0)$ and $B(2, 1)$ is rotated about A in an anticlockwise direction through an angle 15°. Then the equation of the line in new position is

(A) $\sqrt{3}x + y = 1$ (B) $\sqrt{3}x - y = 1$ (C) $x + y = 1$ (D) $x - y = 1$

5. The locus of the point of intersection of the straight lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ (where k is a parameter) is

(A) a parabola (B) a circle (C) a hyperbola (D) an ellipse

6. The line L given by $ax - 3y = 7$ passes through (8, 3). The line L' parallel to L has equation $dx + 5y = d$. The distance between L and L' is

(A) $\frac{5}{\sqrt{13}}$ unit (B) $\frac{7}{\sqrt{13}}$ unit (C) $\sqrt{5}$ unit (D) 1 unit

7. A circle of area 4π square unit has two of its diameters along the lines $x + y = 3$ and $x - y = 1$. The equation of the circle is

(A) $x^2 + y^2 + 4x + 2y + 1 = 0$ (B) $x^2 + y^2 - 4x - 2y - 1 = 0$

(C) $x^2 - y^2 + 4x + 2y + 1 = 0$ (D) $x^2 + y^2 - 4x - 2y + 1 = 0$

8. P is a variable point on a circle C and Q is a fixed point outside C. R is a point on PQ dividing it in ratio $p : q$, where $p > 0$, $q > 0$ are fixed. Then locus of R is

(A) a circle (B) an ellipse (C) a parabola (D) a straight line

9. A line of fixed length $l+m$ moves so that its ends are always on two fixed perpendicular lines. The locus of a point which divides the line into two parts of length l and m is

(A) $\frac{x^2}{l^2}-\frac{y^2}{m^2}=1$

(B) $\frac{x^2}{l^2}+\frac{y^2}{m^2}=1$

(C) $\frac{x^2}{m^2}-\frac{y^2}{l^2}=1$

(D) $\frac{x^2}{m^2}+\frac{y^2}{l^2}=1$

10. The line joining the foci F and F' of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ subtends a right angle at the positive end B of the minor axis. If e is the eccentricity of the ellipse, then e equals to

(A) $\pm\frac{1}{\sqrt{3}}$

(B) $\frac{1}{4}$

(C) $\pm\frac{1}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{2}}$

11. The locus of the centre of a circle which touches two given circles externally is

(A) a parabola

(B) an ellipse

(C) a hyperbola

(D) a circle

12. If the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ meets the curve $\frac{x^2}{1}+\frac{y^2}{a^2}=1$ in exactly two distinct points, then the maximum value of $(a\sin\theta+2\cos\theta)$ is

(A) $-\sqrt{5}$

(B) $\sqrt{5}$

(C) $\frac{1}{\sqrt{5}}$

(D) $-\frac{1}{\sqrt{5}}$

13. The foot of the perpendicular on the line $3x+y=k$ drawn from the origin is C. If the line cuts the x-axis and y-axis at A and B respectively, then $BC:CA$ is

(A) 3 : 1

(B) 9 : 1

(C) 1 : 3

(D) 1 : 9

14. If $4a^2+9b^2-c^2+12ab=0$, then the family of straight lines $ax+by+c=0$ is concurrent at

(A) $(-3, 2)$ or $(3, -2)$

(B) $(-2, 3)$ or $(2, -3)$

(C) $(3, 2)$ or $(-3, 2)$

(D) $(2, 3)$ or $(-2, -3)$

15. The equation $r(\cos 3\theta+\sin 3\theta)=5k\sin\theta\cos\theta$, where $k\in\mathbb{R}\setminus\{0\}$, when transformed to Cartesian coordinates taking pole as the origin and initial line as x-axis, takes the form

(A) $(x+y)(x-2y)^2=5kxy$

(B) $(x-y)(x+2y)^2=5kxy$

(C) $(x-y)(x^2+4xy+y)^2=5kxy$

(D) $(x+y)^2(x-2y)=5kxy$

16. Let S be the set of all positive rational numbers &* be the operation introduce on S by $a*b = \frac{ab}{2}$ or all $a, b \in S$. Then

(A) $(S, *)$ is a semi-group but not a group

(B) $(S, *)$ is an Abelian group

(C) $(S, *)$ is not a semi-group

(D) $(S, *)$ is not an Abelian group

17. Let A, B, C be sets such that $n(A) = 60$, $n(B) = 13$, $n(C) = 27$, $n(A \cap B \cap C) = 4$. Also,

$$n(A \cap B) = n(B \cap C) = n(C \cap A) = \frac{1}{3} n(A \cup B \cup C)$$

Then $n(A \cup B \cup C)$ is

(A) 100 (B) 96 (C) 52 (D) 200

18. The inverse of a skew-symmetric matrix of odd order

(A) is a symmetric matrix

(B) is a diagonal matrix

(C) does not exist

(D) is a skew-symmetric matrix

19. Let $(S, +, \cdot)$ be the ring of all continuous functions in $[0, 1]$, where '+' and '·' denote respectively the addition of functions and multiplication of functions. Then the ring has

(A) both sided zero divisor

(B) left-sided zero divisor but not right-sided

(C) right-sided zero divisor but not left-sided

(D) no zero-divisor

20. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \mathbb{R}$. Let $a + d = 1 = ad - bc$. Then A^3 equals

(A) $\underline{0}$, the null matrix of order 2

(B) $-I_2$

(C) $3I_2$

(D) I_2

I_2 is the identity matrix of order 2.

21. Let $S = \{-1, 0, 1\}$ and '+', '·' denote respectively the addition and multiplication of numbers. Then

(A) $(S, +)$ is a groupoid but $(S, \cdot)$ is not so

(B) $(S, \cdot)$ is a groupoid but $(S, +)$ is not so

(C) both $(S, +)$ and $(S, \cdot)$ are groupoids

(D) neither $(S, +)$ nor $(S, \cdot)$ is a groupoid

22. The ring $\left\{\begin{pmatrix} 2x & 2y \\ 2z & 2t \end{pmatrix} : x, y, z, t \text{ are integer}\right\}$ accompanied by the operations of matrix addition and matrix multiplication is

(A) commutative without identity element

(B) commutative with identity element

(C) non-commutative with identity element

(D) non-commutative without identity element

23. i^i is

(A) purely imaginary

(B) $e^{-\frac{\pi}{2}(4n+1)}$, for any integer n

(C) $e^{-\frac{\pi}{2}(4n+1)} + ie^{-\frac{\pi}{2}(4n)}$, for any integer n

(D) $e^{-\frac{\pi}{2}(1+i)}$

24. For any three sets A, B, C,

$$A \times (B \setminus C) =$$

(A) $A \times B \setminus C$

(B) $(A \setminus C) \times B$

(C) $(A \times B) \cap (A \times C)^C$

(D) $(A \times B) \cup C \setminus (A \times C) \cup B$

25. If z lies in the 1^{st} quadrant such that $|z - 5i| \leq 1$ and amp (z) is minimum, then $z = k(1 + 2\sqrt{6}i)$, where k equals

(A) $\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{2\sqrt{6}}{5}$

(D) $\frac{\sqrt{6}}{5}$

26. Let Z be the sets of all integers and let '$*$' be the binary operation in Z defined by $a*b = a+b+10$, $\forall a, b \in Z$. Then $(Z, *)$ is an Abelian group. The identity element of the group is

(A) -10 (B) 0 (C) 1 (D) 10

27. Let $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ be three vectors of equal magnitude such that each vector is perpendicular to the other and $\vec{\delta} = \vec{\alpha} + \vec{\beta} + \vec{\gamma}$. The angle between $\vec{\alpha}$ and $\vec{\delta}$ is

(A) $\cos^{-1}\frac{1}{\sqrt{3}}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

28. If α is an imaginary root of $x^{11} - 1 = 0$, then $(1-\alpha)(1-\alpha^2)\cdots(1-\alpha^{10}) =$

(A) 0 (B) 11 (C) 10 (D) 1

29. Let G be a non-void set and the binary operation $*$ be introduced in G as follows :

$$a*b = b \text{ for all } a, b \in G$$

Then

(A) $(G, *)$ is a group

(B) $(G, *)$ is a semi-group but not a group

(C) $(G, *)$ is not even a semi-group

(D) $(G, *)$ is a quasi-group but not a semi-group

30. Let $A = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \neq 0 \right\}$ & $*$ be the operation of matrix multiplication. Then

(A) $*$ is not a binary operation

(B) there exists an identity element in the system $(A, *)$

(C) identity element exists but there does not exist inverse of any element

(D) identity element exists in $(A, *)$ if and only if x be an integer

31. Let $\begin{pmatrix} x+y & y-z \\ 5-t & 7+x \end{pmatrix} = \begin{pmatrix} t-x & z-t \\ z-y & x+z+t \end{pmatrix}$. Then

(A) $x=1, y=2, z=3, t=4$ (B) $x=-1, y=3, z=-2, t=9$

(C) $x=1, y=3, z=2, t=5$ (D) $x=0, y=1, z=3, t=4$

32. Let A be any skew-symmetric matrix of order $n(\leq 3)$ over $\mathbb{R}$. Then

(A) det A is never zero

(B) det $A = 0$ if $n = 2$

(C) det $A = 0$ if $n = 3$

(D) det $A > 0$ for all A

33. Let $Z = \left(\frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right)^n$ where n is positive integer. Then mod Z is

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 2

(D) 1

34. Let S be the set of all even integers and T be the set of all odd integers. Let '+' be the usual addition and '.' be the usual multiplication, then

(A) $(S, +, \cdot)$ is a field

(B) $(S, +, \cdot)$ is a ring but not a field

(C) $(T, +, \cdot)$ is a field

(D) $(T, +, \cdot)$ is a ring but not a field

35. Let A be a square matrix of order 3 over $\mathbb{R}$. Then

(A) every orthogonal matrix is non-singular

(B) every non-singular matrix is orthogonal

(C) every square matrix is non-singular

(D) a square matrix of odd order is definitely orthogonal

36. Let z be a complex number such that $\exp(z) = -1$, then

(A) $z = 2n\pi i$

(B) $z = (2n+1)\pi i$

(C) $z = \frac{n\pi i}{2}$

(D) $z = n\pi$

n is integer in all cases.

37. The principal value of argument of $-i$, i.e., $\arg(-i)$ is

(A) $\frac{3\pi}{2}$

(B) $-\frac{3\pi}{2}$

(C) $-\frac{\pi}{2}$

(D) $\frac{\pi}{2}$

38. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \neq \vec{0}$. Then

(A) $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{b} = \vec{c}$

(B) $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{b} = \vec{c}$

(C) $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{b} = \vec{c}$

(D) $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0 \Rightarrow \vec{b} = \vec{c}$

39. Let a, b, c, d satisfy the equations

$$a + 7b + 3c + 5d = 0$$
$$8a + 4b + 6c + 2d = -16$$
$$2a + 6b + 4c + 8d = 16$$
$$5a + 3b + 7c + d = -16$$

Then $(a+d)(b+c)$ is equal to

(A) 16 (B) −16 (C) 0 (D) 8

40. Let a, b be two positive real numbers and $A = \begin{pmatrix} a & 1 \\ 2 & b \end{pmatrix}$. Then the number of real roots of the equation in λ, $\det(A - \lambda I_2) = 0$ (I_2 is the identity matrix of order 2) is

(A) 0 (B) 1 (C) 2 (D) 3

41. The sum of 99^{th} powers of the roots of $x^7 - 1 = 0$

(A) 99 (B) −1 (C) 7 (D) 0

42. Let three distinct positive real numbers a, b, c are in GP and x be a real number such that $a + b + c = xb$. Then

(A) $-3 < x < 1$

(B) $x > 1$ or $x < -3$

(C) $x < -1$ or $x > 3$

(D) $-1 < x < 3$

43. The coefficient of y^{2n-2} in $f(y) = (y-1)(y+1)(y-2)(y+2)\cdots(y-n)(y+n)$ is

(A) n^{2n-2}

(B) 1

(C) $\frac{n(n+1)(2n+1)}{6}$

(D) $-\frac{n(n+1)(2n+1)}{6}$

44. For the non-void sets A, X, Y;

(A) $A \cap X = A \cap Y \Rightarrow X = Y$

(B) $A \cup X = A \cup Y \Rightarrow X = Y$

(C) $A \cap X = A \cap Y$ and $A \cup X = A \cup Y \Rightarrow X = Y$

(D) $A - X = A - Y \Rightarrow X = Y$

45. Let A be a square matrix of order 3 with $\det A = 5$. If $B = 4A^2$, then $\det B$ is equal to

(A) 20 (B) 100 (C) 320 (D) 1600

46. If the normal at the point $\left(ct_1, \frac{c}{t_1}\right)$ meets the curve $xy = c^2$ again at the point $\left(ct_2, \frac{c}{t_2}\right)$, then

(A) ${t_1}^2 t_2 = -1$ (B) ${t_2}^2 t_1 = -1$ (C) $t_2 t_1^3 = -1$ (D) $t_1 t_2^3 = -1$

47. The equation(s) of the tangent(s) to the ellipse $2x^2 + 3y^2 = 1$ which are parallel to the line $2x - y + 3 = 0$ is

(A) $2x - y \pm 7 = 0$

(B) $y = 2x \pm \sqrt{\frac{7}{3}}$

(C) $y = 2x \pm 13$

(D) $y = 2x \pm \frac{3}{\sqrt{13}}$

48. Let $f : [a, b] \rightarrow \mathbb{R}$ be continuous in $[a, b]$ and $f(a) = f(b) = 0$. Then

(A) if f be derivable in $[a, b]$, then there is at least one point $c \in [a, b]$ such that $f'(c) = 0$

(B) if f be derivable in $[a, b]$, then there exists at least one point $c \in (a, b)$ such that $f'(c) = 0$

(C) if there exists $c \in (a, b)$ such that $f'(c) = 0$, then f must satisfy the conditions of Rolle's theorem

(D) Rolle's theorem is true only for polynomial functions of one variable

49. For the differentiable function f, if $f(0) = 5$ and $f(x) < 5$ for $x \neq 0$, then

(A) $f'(0) = 0$ (B) $f'(0) = 5$ (C) $f'(0) = -5$ (D) $f'(0) = 1$

50. Let $f(x) = (1-x)^{3/2}$ and $f(1) = f(0) + f'(0) + \frac{1}{2!} f''(\theta)$, $0 < \theta < 1$. The value of θ is

(A) $\frac{16}{9}$ (B) $\frac{9}{16}$ (C) $\frac{3}{4}$ (D) $\frac{7}{16}$

51. Let $f(x) = \begin{cases} x^2 + x + 1, & 0 \le x \le 1 \\ 2x+1, & 1 \le x \le 2 \end{cases}$. Then at $x = 1$, f is

(A) continuous but not differentiable

(B) differentiable

(C) not continuous because limit does not exist

(D) not continuous but limit exists

52. The value of $\lim_{x \to 3} \frac{f(x)(x+3) - 12}{(x-3)}$ when $f(3) = 2$ and $f'(3) = 5$ is

(A) 30 (B) 32 (C) 62 (D) 60

53. The maximum value of $\left(\frac{1}{x+1}\right)^{x+1}$ is

(A) $(1+e)^{1/e}$ (B) $e^{1/e}$ (C) $(1+e)^{1/(1+e)}$ (D) $e^{\frac{1}{e}+1}$

54. If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, then $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} =$

(A) u (B) 0 (C) $2u$ (D) xy

55. The value of e^{π} is

(A) strictly greater than π^e

(B) strictly less than π^e

(C) strictly less than $2^{\pi e/2}$

(D) equal to π^e

56. $\int \sin^2 x \cos^5 x \, dx$ is

(A) $\frac{\sin^7 x}{7} - \frac{\sin^5 x}{5} + \frac{\sin^3 x}{3} + C$

(B) $\frac{\sin^7 x}{7} - \frac{2\sin^5 x}{5} + \frac{\sin^3 x}{3} + C$

(C) $\frac{\sin^7 x}{7} + \frac{\sin^5 x}{5} - \frac{\sin^3 x}{3} + C$

(D) $\frac{\sin^7 x}{7} + \frac{2\sin^5 x}{5} + \frac{\sin^3 x}{3} + C$

C is constant of integration.

57. The value of $\Gamma\left(\frac{7}{2}\right)$ is

(A) $\frac{15}{8}\sqrt{\pi}$ (B) $\frac{105}{16}\sqrt{\pi}$ (C) $\frac{15}{8}\pi$ (D) $\frac{105}{16}\pi$

58. $\lim_{n\to\infty}\left\{\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}\right\}$ is

(A) 1 (B) $\tan^{-1}(1)$ (C) 0 (D) $\log(2)$

59. The solution of the differential equation $\frac{dy}{dx} = \cos^2\left(\frac{2y}{x}\right) + \frac{y}{x}$, subject to the condition $y = \pi$ when $x = 1$, is

(A) $\frac{1}{2}\tan\left(\frac{2y}{x}\right) = \log(x)$ (B) $\tan\left(\frac{2y}{x}\right) = 2\log(x) + 1$

(C) $\frac{1}{2}\sin\left(\frac{4y}{x}\right) + 1 = \log(x)$ (D) $\frac{1}{2}\sin\left(\frac{4y}{x}\right) = \log(x)$

60. The solution of $(D^2 + D)y = 0\left(D \equiv \frac{d}{dx}\right)$ is

(A) $y = A\log(x) + B$ (B) $y = Ae^{-x} + B$

(C) $y = Ae^{x} + B$ (D) $y = (Ax + B)e^{-x}$

61. The values of a and b for which $\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3} = 1$ are

(A) $a=\frac{3}{2}, b=\frac{5}{2}$ (B) $a=-\frac{5}{2}, b=-\frac{3}{2}$

(C) $a=-\frac{3}{2}, b=-\frac{5}{2}$ (D) $a=\frac{3}{2}, b=-\frac{5}{2}$

62. $\int_0^\infty 5^{-x^2}\,dx =$

(A) $\frac{\sqrt{\pi}}{\sqrt{\ln 5}}$ (B) $\frac{\sqrt{\pi}}{2\sqrt{\ln 5}}$ (C) $\frac{1}{2\sqrt{\ln 5}}$ (D) $\frac{1}{\sqrt{\ln 5}}$

63. Let $f(x)=\begin{cases} k, & 0 \le x \le k \\ 2k, & k < x \le 1 \end{cases}$. Then the value of k for which $\int_0^1 f(x)\,dx = \frac{7}{16}$ is

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

64. Suppose $\omega = x^3 + yz + xy + zx$ and $x = 3\cos t$, $y = 3\sin t$ and $z = 2t$. Then the value of $\frac{d\omega}{dt}$ at $t=\frac{\pi}{2}$ is

(A) 0 (B) $-3(\pi+1)$ (C) -3π (D) $-\pi$

65. The equation $y-2x=a$ represents the orthogonal trajectories of the family

(A) $y=ce^x$ (B) $y^2=x^2+c$ (C) $2y+x=c$ (D) $x+y=ce^x$

where a is arbitrary constant.

66. Let $f(x)=\begin{cases} [x]+5, & x \le 2 \\ kx^2, & x > 2 \end{cases}$. Then the value of k which makes f continuous at $x=2$ is

(A) $\frac{3}{2}$ (B) $\frac{7}{4}$ (C) $\frac{7}{2}$ (D) 3

67. Let $\int_a^b f(x)\,dx = (b-a) f\{a + \theta(b-a)\}$ (for some $0 < \theta < 1$) and $f(x) = 2x + k$. Then

(A) $\theta = \frac{1}{2}$ (B) $\theta = \frac{1}{4}$ (C) $\theta = \frac{3}{4}$ (D) $\theta = \frac{1}{4}, \frac{1}{2}$

68. Let $f : R \to \mathbb{R}$ be such that $f'''(x)$ exists. Suppose $f(a) = f(b) = f'(a) = f'(b) = 0$ for some a, b; $a < b$. Then

(A) $f'''(x) > 0$ in $[a, b]$

(B) $f'''(x) < 0$ in $[a, b]$

(C) $f'''(c) = 0$ for some $c \in (a, b)$

(D) $f'''(x) > 0$ in $\left(a, \frac{a+b}{2}\right)$ and $f'''(x) < 0$ in $\left(\frac{a+b}{2}, b\right)$

69. Let $I = \int_0^{50\pi} \sqrt{(1 - \cos 2x)}\,dx$. Then

(A) $I = 200\sqrt{2}$ (B) $I = 100\sqrt{2}$ (C) $I = 1$ (D) $I = 2$

70. Let $u(x, y) = \ln \frac{x^3 + y^3}{x + y}$. Then $xu_x + yu_y$

(A) does not exist

(B) is $\ln 2$

(C) is 2

(D) is $\frac{1}{2}$

71. Two linearly independent solutions of the differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$ are

(A) x^2 and $x \log x$

(B) x^2 and $\log x$

(C) x and $\log x$

(D) x^2 and $x^2 \log x$

72. General solution of the differential equation $\dfrac{d^2y}{dx^2} + y = xe^{2x}$ is

(A) $y = c_1 \cos x + c_2 \tan x + \dfrac{e^{2x}}{25}(5x-4)$

(B) $y = c_1 \cos x + c_2 \sin x + e^{x}$

(C) $y = c_1 \cos x + c_2 \sin x + \dfrac{e^{2x}}{25}(5x-4)$

(D) $y = c_1 \cos x + c_2 \tan x + e^{-x}$

where c_1 and c_2 are arbitrary constants.

73. If $f''(x)$ is continuous in $[a, a+h]$ and $f''(a) \neq 0$, then in the relation

$$f(a+h) = f(a) + hf'(a+\theta h),\ 0 < \theta < 1$$

then the value of $\lim\limits_{h \to 0} (\theta)$ is

(A) 0 (B) 1 (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{2}$

74. The value of $\lim\limits_{x \to 2} (5x-9)^{\frac{1}{(x-2)}}$ is

(A) 1 (B) e^5 (C) $e^{1/5}$ (D) $+\infty$

75. The values of $f'(2)$ and $f'(3)$ where $f(x) = 2|x-1| + 5|x-4|$ are respectively

(A) 12, 9 (B) 3, 3 (C) −3, −3 (D) −8, −1

76. For the curve $(x^2 + y^2)^2 = a^2x^2 - b^2y^2,\ ab \neq 0$

(A) there is no tangent at the origin

(B) the tangent at origin is $x = 0$

(C) the tangent at origin is $y = 0$

(D) tangents at origin are $ax \pm by = 0$

77. Let $f : \mathbb{R} \to \mathbb{R}$ be an even function. Then

(A) $f'(x)$ always exists at all points of $\mathbb{R}$

(B) if $f'(x)$ exists at all x, then $f'(0) = 0$

(C) $f'(x)$ is never zero

(D) $f'(x)$ is always greater than zero

78. Let $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then

(A) $f'(x)$ exists at all points of $\mathbb{R}$

(B) $f'(x)$ does not exist at $x = 0$

(C) $f(x)$ is discontinuous at $x = 0$ only

(D) $f(x)$ is discontinuous everywhere

79. Let $I = \int_0^1 \frac{dx}{(1-x)^{\mu}}$, $\mu > 0$. Then

(A) the integrand is continuous and so I exists

(B) the integrand has removable discontinuity and so I exists for all μ

(C) the integrand has infinite discontinuity at $x = 1$ only and I exists for all $\mu < 1$ only

(D) the integral I never exists

80. $\int_{-a}^{a} \frac{x^3 \sin(1+x^2)}{1+x^2} dx$ is

(A) a (B) 0 (C) $2a$ (D) 1

81. $\int \frac{\tan(\sqrt{x})\sec^2(\sqrt{x})}{\sqrt{x}} dx$ is

(A) $\sec(\sqrt{x})\tan(\sqrt{x}) + c$

(B) $\tan^2(\sqrt{x}) + c$

(C) $\sec(\sqrt{x}) + c$

(D) $\sin(\sqrt{x}) + \cos(\sqrt{x}) + c$

82. Let $f(x, y) = \begin{cases} \dfrac{x^3 y}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$ and f_{xy} is defined as $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial y}\right)$. Then

(A) $f_{yx}(0, 0) = 0$, $f_{xy}(0, 0) = 1$

(B) $f_{xy}(0, 0) = 0$, $f_{yx}(0, 0) = 1$

(C) $f_{xy}(0, 0) = 0 = f_{yx}(0, 0)$

(D) neither f_{xy} nor f_{yx} exists at (0, 0)

83. Given that $\dfrac{x^2}{a^2+u} + \dfrac{y^2}{b^2+u} = 1$, where $u \equiv u(x, y)$, a differentiable function of x and y. Then

(A) $u_x^2 + u_y^2 = 2(xu_x + yu_y)$

(B) $u_x^2 + u_y^2 = u_{xy} + u_{yx}$

(C) $u_x^2 + u_y^2 \neq 2(xu_x + yu_y)$

(D) $u_x^2 + u_y^2 = 4u_{xy}$

84. Let $f(x, y) = |x|(1+y)$, $(x, y) \in \mathbb{R}^2$. Then

(A) both f_x and f_y exist at (0, 0)

(B) f_x exists at (0, 0) but f_y does not

(C) f_x does not exist at (0, 0) but f_y exists at (0, 0)

(D) neither f_x nor f_y exists at (0, 0)

85. In the Taylor's expansion of $f(x) = e^{x/2}$ about $x = 3$, the coefficient of $(x-3)^5$ is

(A) $e^{-3/2}\dfrac{1}{5!}$

(B) $e^{-3/2}\dfrac{1}{2^5 . 5!}$

(C) $e^{3/2}\dfrac{2^5}{5!}$

(D) $e^{3/2}\dfrac{1}{2^5 . 5!}$

86. Arithmetic mean and standard deviation of a Binomial distribution $B(n, p)$ are respectively 4 and $\sqrt{\dfrac{8}{3}}$. The values of p and n are

(A) $\dfrac{2}{3}$, 12

(B) $\dfrac{1}{3}$, 12

(C) $\dfrac{1}{4}$, 12

(D) $\dfrac{1}{\sqrt{6}}$, 12

87. Consider the frequency distribution

x	0	1	2	3	4
f	7	14	23	26	10

The mean and the median are respectively

(A) 2, 2

(B) 2·225, 1

(C) 2·225, 2

(D) 2, 1

88. A card is drawn from each of two well-shuffled packs of cards. The probability that at least one of them is a King is

(A) $\frac{2}{13}$ (B) $\frac{1}{13}$ (C) $\frac{25}{169}$ (D) $\frac{1}{26}$

89. Consider the frequency table

Value	Less than 10	Less than 20	Less than 30	Less than 40
Frequency	4	16	40	50

The mode is

(A) 23·75 (B) 28·57 (C) 23 (D) 28

90. The standard deviation of n data $x_1, x_2, \ldots, x_n$ is 27. Then the standard deviation of $3x_i + 5$, $i = 1, 2, \ldots, n$ is

(A) 81 (B) 9 (C) 86 (D) 4

91. A discrete random variable X has the following probability mass function :

x	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

The value of k is

(A) 1 or $\frac{1}{10}$ (B) $\frac{1}{10}$ (C) -1 (D) $\frac{1}{10}$ or -1

92. The value of k for which $f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0 \quad , & \text{otherwise} \end{cases}$ is a probability density function, is

(A) 1 (B) 3 (C) 6 (D) 4

93. Two squares are chosen at random from the small squares drawn on a chess board. The probability that the two squares chosen have exactly one corner in common is

(A) $\frac{7}{144}$ (B) $\frac{7}{72}$ (C) $\frac{7}{36}$ (D) $\frac{7}{18}$

94. The probability of selecting a black card or a 6 from a deck of 52 cards is

(A) $\frac{1}{13}$ (B) $\frac{2}{5}$ (C) $\frac{1}{5}$ (D) $\frac{7}{13}$

95. The following table gives incomplete frequency distribution :

Class value	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	5	12	X	20	Y	10	4

It is known that total frequency is 100 and the median is 44. Then the mode is

(A) 37·98 (B) 37·22 (C) 35·10 (D) 34·20

96. The correlation coefficient between x and y is 0·5. The correlation coefficient between $5x$ and $-3y$ is

(A) 0·5 (B) −0·5 (C) 0·3 (D) −0·3

97. A committee of 3 persons is to be formed from 4 male persons and 3 female persons taking at least 1 male person and 1 female person. The number of ways of formation of such committee is

(A) 60 (B) 45 (C) 30 (D) 216

98. The number of ways that the letters of the word TREES can be arranged such that each word starts with a consonant and ends with a vowel is

(A) 9 (B) 18 (C) 24 (D) 27

99. The letters of the word FAILURE are arranged in such a way that the vowels are never separated. Then the total number of such words is

(A) 24 (B) 576 (C) 120 (D) 96

100. Let P_n be an n-sided regular polygon in a plane. Let T_n be the number of all possible triangles formed by joining the vertices of P_n. If $T_{n+1} - T_n = 10$, then

(A) $n = 5$ (B) $n = 10$ (C) $n = 7$ (D) $n = 8$

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