WBJEE - 2020

Answer Keys by

Aakash Institute, Kolkata Centre

MATHEMATICS

Q.No.	✿	✚	◯	☐
01	A	D	*	D
02	A	B	B	B
03	C	B	C	B
04	D	A	C	D
05	B	A	C	B
06	D	B	C	C
07	C	D	A	B
08	C	C	B	D
09	C	D	A	D
10	B	*	A	B
11	C	B	C	B
12	A	C	D	A
13	A	C	B	A
14	A	C	D	B
15	A	C	C	D
16	B	A	C	C
17	C	B	C	D
18	D	A	B	*
19	B	A	C	B
20	A	C	A	C
21	B	D	A	C
22	D	B	A	C
23	C	D	A	C
24	A	C	B	A
25	D	C	C	B
26	D	C	D	A
27	B	B	B	A
28	B	C	A	C
29	D	A	B	D
30	B	A	D	B
31	C	A	C	D
32	B	A	A	C
33	D	B	D	C
34	D	C	D	C
35	B	D	B	B
36	B	B	B	C
37	A	A	D	A
38	A	B	B	A
39	B	D	C	A
40	D	C	B	A
41	C	A	D	B
42	D	D	D	C
43	*	D	B	D
44	B	B	B	B
45	C	B	A	A
46	C	D	A	B
47	C	B	B	D
48	C	C	D	C
49	A	B	C	A
50	B	D	D	D
51	C	C	B	D
52	B	B	B	B
53	C	C	C	C
54	C	B	B	C
55	C	B	C	B
56	A	C	C	C
57	D	B	C	B
58	D	C	A	B
59	B	C	D	C
60	C	C	D	C
61	C	A	B	C
62	B	D	C	C
63	C	D	C	C
64	B	B	B	A
65	B	C	C	D
66	B	D	C	C,D
67	A,D	A	A,C	A,B
68	A,C	C	B	D
69	B	A,C	A,D	A
70	C,D	B	A,C	C
71	A,B	A,D	B	A,C
72	D	A,C	C,D	B
73	A	B	A,B	A,D
74	C	C,D	D	A,C
75	A,C	A,B	A	B

*** No correct option**

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Code - ✿

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ANSWERS & HINT
for
WBJEE - 2020
SUB : MATHEMATICS

CATEGORY - I (Q1 to Q50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

1. Let $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$. Then

(A) $x^2y_2 + xy_1 + n^2y = 0$ (B) $xy_2 - xy_1 + 2n^2y = 0$ (C) $x^2y_2 + 3xy_1 - n^2y = 0$ (D) $xy_2 + 5xy_1 - 3y = 0$

Ans : (A)

Hint : $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n = n \times \log\left(\frac{x}{n}\right)$

$\Rightarrow -\frac{1}{\sqrt{b^2 - y^2}}.y_1 = \frac{n}{x}$

$\Rightarrow x^2\, y_1^{\,2} = n^2(b^2 - y^2) \Rightarrow x^2y_2 + xy_1 + n^2y = 0$

2. Let $\varphi(x) = f(x) + f(1-x)$ and $f''(x) < 0$ in [0, 1], then

(A) φ is monotonic increasing in $\left[0, \frac{1}{2}\right]$ and monotonic decrasing in $\left[\frac{1}{2}, 1\right]$

(B) φ is monotonic increasing in $\left[\frac{1}{2}, 1\right]$ and monotonic decrasing in $\left[0, \frac{1}{2}\right]$

(C) φ is neither increasing nor decreasing in any sub interval of [0, 1]

(D) φ is increasing [0, 1]

Ans : (A)

Hint : $\phi'(x) = f'(x) - f'(1 - x)$

$f'(x) - f'(1 - x) \geq 0$ (for monotonic increasing)

$f'(x) \geq f'(1 - x),\ x \leq 1 - x$ ($\because f'(x)$ is decreasing)

$x \leq \frac{1}{2} \Rightarrow \phi(x)$ is monotonic increasing in $\left[0, \frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2}, 1\right]$

3. $\int \frac{f(x)\varphi'(x)+\varphi(x)f'(x)}{(f(x)\varphi(x)+1)\sqrt{f(x)\varphi(x)-1}}dx =$

(A) $\sin^{-1}\sqrt{\frac{f(x)}{\varphi(x)}}+c$

(B) $\cos^{-1}\sqrt{(f(x))^2-(\varphi(x))^2}+c$

(C) $\sqrt{2}\tan^{-1}\sqrt{\frac{f(x)\varphi(x)-1}{2}}+c$

(D) $\sqrt{2}\tan^{-1}\sqrt{\frac{f(x)\varphi(x)+1}{2}}+c$

Ans : (C)

Hint : Let f(x) ϕ(x) = t

$$\int \frac{dt}{(t+1)\sqrt{t-1}}$$

Let $t - 1 = p^2$, $dt = 2p\,dp$

$$\Rightarrow \int \frac{2dp}{p^2+2} = \sqrt{2}\tan^{-1}\left|\sqrt{\frac{f(x)\phi(x)-1}{2}}\right|+c$$

4. The value of $\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x\,dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$ is equal to

(A) 27 (B) 54 (C) –54 (D) 0

Ans : (D)

Hint : $-\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}xdx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}xdx$

= 0

5. $\int_0^2 [x^2]dx$ is equal to

(A) 1 (B) $5-\sqrt{2}-\sqrt{3}$ (C) $3-\sqrt{2}$ (D) 8/3

Ans : (B)

Hint : $\int_0^1 0dx+\int_1^{\sqrt{2}}1.dx+\int_{\sqrt{2}}^{\sqrt{3}}2.dx+\int_{\sqrt{3}}^{2}3dx$

$= 5-\sqrt{2}-\sqrt{3}$

6. If the tangent to the curve $y^2 = x^3$ at (m^2, m^3) is also a normal to the curve at (M^2, M^3), then the value of mM is

(A) $-\frac{1}{9}$ (B) $-\frac{2}{9}$ (C) $-\frac{1}{3}$ (D) $-\frac{4}{9}$

Ans : (D)

Hint : $2yy_1 = 3x^2$

$$y_1=\frac{3x^2}{2y}\Rightarrow (y_1)_{m^2,\,m^3}=\frac{3\times m^4}{2\times m^3}=\frac{3m}{2}$$

Again ; slope of normal = $-\frac{2}{3M}$, mM = $-\frac{4}{9}$

7. If $x^2+y^2=a^2$, then $\int_0^a \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx =$

(A) $2\pi a$ (B) πa (C) $\frac{1}{2}\pi a$ (D) $\frac{1}{4}\pi a$

Ans : (C)

Hint : $y_1 = -\frac{x}{y}$

$$\int_0^a \sqrt{1+\frac{x^2}{y^2}}\,dx = a\int_0^a \frac{1}{y}.dx = a\int_0^a \frac{dx}{\sqrt{a^2-x^2}}$$

$$= a\left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a = a\pi/2$$

8. Let f, be a continuous function in [0, 1], then $\lim_{n\to\infty}\sum_{j=0}^{n}\frac{1}{n}f\left(\frac{j}{n}\right)$ is

(A) $\frac{1}{2}\int_0^{\frac{1}{2}} f(x)dx$ (B) $\int_{\frac{1}{2}}^{1} f(x)dx$ (C) $\int_0^1 f(x)dx$ (D) $\int_0^{\frac{1}{2}} f(x)dx$

Ans : (C)

Hint : $\int_0^1 f(x)\,dx$

9. Let f be a differentiable function with $\lim_{x\to\infty} f(x)=0$. If $y' + yf'(x) - f(x)f'(x) = 0$, $\lim_{x\to\infty} y(x) = 0$, then $\left(\text{where } y' = \frac{dy}{dx}\right)$

(A) $y + 1 = e^{f(x)} + f(x)$ (B) $y - 1 = e^{f(x)} + f(x)$ (C) $y + 1 = e^{-f(x)} + f(x)$ (D) $y - 1 = e^{-(f)x} + f(x)$

Ans : (C)

Hint : $\frac{dy}{dx} + f'(x)y = f'(x)f(x)$

$\Rightarrow y \times e^{f(x)} = \int f'(x)f(x)e^{f(x)}dx$

$\Rightarrow y \times e^{f(x)} = e^{f(x)}(f(x)-1)+c$ [Putting f(x) = 0 ; y = 0, c = 1]

$\Rightarrow y \times e^{f(x)} = e^{f(x)}(f(x)-1)+1$

$\Rightarrow y = f(x) - 1 + e^{-f(x)}$

$\Rightarrow y + 1 = e^{-f(x)} + f(x)$

10. If $x\sin\left(\frac{y}{x}\right)dy = \left[y\sin\left(\frac{y}{x}\right) - x\right]dx$, $x > 0$ and $y(1) = \frac{\pi}{2}$ then the value of $\cos\left(\frac{y}{x}\right)$ is

(A) 1 (B) logx (C) e (D) 0

Ans : (B)

Hint : $\sin v\left(v + x.\frac{dv}{dx}\right) = v\sin v - 1$

$\Rightarrow x\sin v.\frac{dv}{dx} = -1$

$\Rightarrow \int \sin v\, dv = -\int \frac{dx}{x}$

$\Rightarrow -\cos v = -\log x + c$

at x = 1 ; y = $\frac{\pi}{2}$; c = 0

$\cos\left(\frac{y}{x}\right) = \log x$

11. Let $f(x) = 1-\sqrt{(x^2)}$ where the square root is to be taken positive, then

(A) f has no extrema at x = 0
(B) f has minima at x = 0
(C) f has maxima at x = 0
(D) f' exists at 0

Ans : (C)

Hint : f(x) = 1 – |x|, f has maxima at x = 0

12. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1[a > 0]$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a is equal to

(A) 2
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) 3

Ans : (A)

Hint : $f'(x) = 6x^2 - 18ax + 12a^2 \Rightarrow f''(x) = 12x - 18a \Rightarrow f'(x) = 0 \Rightarrow x = a, 2a$

$f''(a) < 0$; p = a (maximum)

$f''(2a) > 0$; q = 2a (minimum)

$a^2 = 2a$; a(a – 2) = 0, a = 2

13. If a and b are arbitrary positive real numbers, then the least possible value of $\frac{6a}{5b}+\frac{10b}{3a}$ is

(A) 4
(B) $\frac{6}{5}$
(C) $\frac{10}{3}$
(D) $\frac{68}{15}$

Ans : (A)

Hint : $\frac{6a}{5b}+\frac{10b}{3a} \ge 2\sqrt{\frac{6a}{5b}\times\frac{10b}{3a}}$, $\frac{6a}{5b}+\frac{10b}{3a} \ge 2\times 2 \ge 4$

14. If $2\log(x+1) - \log(x^2-1) = \log 2$, then x =

(A) only 3
(B) –1 and 3
(C) only –1
(D) 1 and 3

Ans : (A)

Hint : $\log\left\{\frac{(x+1)^2}{x^2-1}\right\} = \log 2 \Rightarrow (x+1)^2 = 2(x^2-1) \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$

$x = 3$; $x \ne -1$

15. The number of complex numbers p such that |p| = 1 and imaginary part of p^4 is 0, is

(A) 4 (B) 2 (C) 8 (D) infinitely many

Ans : (A)

Hint : Let p = x + iy, $p^2 = (x^2 - y^2) + 2ixy$, $p^4 = (x^2 - y^2)^2 - 4x^2y^2 + 4ixy(x^2 - y^2)$

Now, $xy(x^2 - y^2) = 0$, $x = \pm y \Rightarrow y^2 = \frac{1}{2} \Rightarrow y = \pm\frac{1}{\sqrt{2}}$

Four complex numbers .

16. The equation $z\bar{z} + (2-3i)z + (2+3i)\bar{z} + 4 = 0$ represents a circle of radius

(A) 2 unit (B) 3 unit (C) 4 unit (D) 6 unit

Ans : (B)

Hint : Centre and radius of $z\bar{z} + \bar{a}z + a\bar{z} + b = 0$ are –a and $\sqrt{|a|^2 - b}$ $\therefore$ radius $= \sqrt{13-4} = 3$

17. The expression $ax^2 + bx + c$ (a, b and c are real) has the same sign as that of a for all x if

(A) $b^2 - 4ac > 0$ (B) $b^2 - 4ac \neq 0$

(C) $b^2 - 4ac \leq 0$ (D) b and c have the same sign as that of a

Ans : (C)

Hint : C-I: If a > 0, $ax^2 + bx + c > 0$, $b^2 - 4ac < 0$, C-II: If $a \leq 0$, $ax^2 + bx + c \leq 0$, $D \leq 0$

18. In a 12 storied building, 3 persons enter a lift cabin. It is known that they will leave the lift at different floors. In how many ways can they do so if the lift does not stop at the second floor ?

(A) 36 (B) 120 (C) 240 (D) 720

Ans : (D)

Hint : Total no. of ways $= {10}_{p_3} = 720$ (except the floor they enter and second floor)

19. If the total number of m-element subsets of the set A = {a_1, a_2, ... a_n} is k times the number of m element subsets containing a_4, then n is

(A) (m – 1)k (B) mk (C) (m + 1)k (D) (m + 2)k

Ans : (B)

Hint : $n_{c_m} = k \cdot {}^{n-1}c_{m-1} \Rightarrow n = mk$

20. Let I(n) = n^n, J(n) = 1.3.5 ..... (2n – 1) for all (n > 1), n $\in$ N, then

(A) I(n) > J(n) (B) I(n) < J(n) (C) I(n) = J(n) (D) $I(n) = \frac{1}{2}J(n)$

Ans : (A)

Hint : AM $\geq$ GM

$\frac{1+3+5+7+...+(2n-1)}{n} > (J(n))^{\frac{1}{n}}$, $\frac{n^2}{n} > (J(n))^{\frac{1}{n}}$, $n^n > J(n)$, $I(n) > J(n)$

21. If c_0, c_1, c_2, ... c_{15} are the Binomial co-efficients in the expansion of $(1 + x)^{15}$, then the value of $\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \cdots\cdots + 15\frac{c_{15}}{c_{14}}$ is

(A) 1240 (B) 120 (C) 124 (D) 140

Ans : (B)

Hint : $S_n = \sum_{r=1}^{15} r\frac{{}^{15}c_r}{{}^{15}c_{r-1}} = \sum_{r=1}^{15}(15 - r + 1) = 16 \times 15 - \frac{15\times16}{2} = 120$

22. Let $A = \begin{pmatrix} 3-t & 1 & 0 \\ -1 & 3-t & 1 \\ 0 & -1 & 0 \end{pmatrix}$ and det A = 5, then

(A) t = 1 (B) t = 2 (C) t = –1 (D) t = –2

Ans : (D)

Hint : $|A| = \begin{vmatrix} 3-t & 1 & 0 \\ -1 & 3-t & 1 \\ 0 & -1 & 0 \end{vmatrix} = 3 - t = 5, t = -2$

23. Let $A = \begin{pmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{pmatrix}$. The value of x for which the matrix A is not invertible is

(A) 6 (B) 12 (C) 3 (D) 2

Ans : (C)

Hint : If matrix is not invertible $\Rightarrow |A| = 0$

$\therefore |A| = \begin{vmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{vmatrix} = 0 \Rightarrow x = 3$

24. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2 × 2 real matrix with det A = 1. If the equation det $(A - \lambda I_2) = 0$ has imaginary roots (I_2 be the Identity matrix of order 2), then

(A) $(a + d)^2 < 4$ (B) $(a + d)^2 = 4$ (C) $(a + d)^2 > 4$ (D) $(a + d)^2 = 16$

Ans : (A)

Hint : |A| = 0 $\therefore$ ad – bc = 1

$|A - \lambda I_2| = 0$

$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$\therefore ad - (a + d)\lambda + \lambda^2 - bc = 0$

$\lambda^2 - (a + d)\lambda + 1 = 0$

$\therefore (a + d)^2 < 4$

25. If $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = ka^2b^2c^2$, then k =

(A) 2 (B) –2 (C) –4 (D) 4

Ans : (D)

Hint : $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = (abc) \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

opening through R –1 = $4a^2b^2c^2$

26. If $f : S \to \mathbb{R}$ where S is the set of all non-singular matrices of order 2 over $\mathbb{R}$ and $f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]$ = ad – bc, then

(A) f is bijective mapping
(B) f is one-one but not onto
(C) f is onto but not one-one
(D) f is neither one-one nor onto

Ans : (D)

Hint : $f\left[\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right] = 4 = f\left[\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\right]$

$\Rightarrow$ not one-one

As $0 \in \mathbb{R}$ but S does not contain any singular matrix so, f is not onto

27. Le the relation ρ be defined on $\mathbb{R}$ by a ρ b holds if and only if a – b is zero or irrational, then

(A) ρ is equivalence relation
(B) ρ is reflexive & symmetric but is not transitive
(C) ρ is reflexive and transitive but is not symmetric
(D) ρ is reflexive only

Ans : (B)

Hint : If a – b = 0 then b – a = 0,

if a – b is irrational then b – a is irrational

$\therefore$ a ρ b $\Rightarrow$ b ρ a $\Rightarrow$ symmetric

$\forall a \in \mathbb{R}$, a – a = 0 a ρ a $\Rightarrow$ reflexive

If a = 2, b = $\sqrt{2}$, c = 3, then

a ρ b, b ρ c but a ρ c is not true $\Rightarrow$ not transitive

28. The unit vector in ZOX plane, making angles 45° and 60° respectively with $\vec{\alpha} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{j} - \hat{k}$ is

(A) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$
(B) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$
(C) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$
(D) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$

Ans : (B)

Hint : Let the vector be $\vec{r} = x\hat{i} + z\hat{k} \Rightarrow |\vec{r}| = 1$

$\vec{r} \cdot \vec{\alpha} = |\vec{r}||\vec{\alpha}| \cos 45°$

$\therefore 2x - z = \frac{3}{\sqrt{2}}$

$\vec{r} \cdot \vec{\beta} = |\vec{r}||\vec{\beta}| \cos 60°$

$z = -\frac{1}{\sqrt{2}}$

$\therefore x = \frac{1}{\sqrt{2}}$

$\therefore \vec{r} = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$

29. Four persons A, B, C and D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game. What is the probability that A wins if A begins ?

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{7}{12}$
(D) $\frac{8}{15}$

Ans : (D)

Hint : P(A win) = $\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)+\cdots$

$= \dfrac{\frac{1}{2}}{1-\left(\frac{1}{2}\right)^4} = \dfrac{\frac{1}{2}}{\frac{15}{16}} = \dfrac{8}{15}$

30. A rifleman is firing at a distant target and has only 10% chance of hitting it. The least number of rounds he must fire to have more than 50% chance of hitting it at least once, is

(A) 5 (B) 7 (C) 9 (D) 11

Ans : (B)

Hint : P(hitting a target) = $\frac{1}{10}$

$\therefore$ P(not hitting a target) = $\frac{9}{10}$

$\therefore$ Let number of trials = n

So, P(hitting at least once) = 1 – P (missing all) = 1 – $\left(\frac{9}{10}\right)^n \geq \frac{1}{2}$

$\Rightarrow (0.9)^n \leq 0.5$

$(0.9)^6 = 0.531441$, $(0.9)^7 = 0.4782969 \Rightarrow n = 7$

31. cos (2x + 7) = a(2 – sin x) can have a real solution for

(A) all real values of a (B) $a \in [2, 6]$ (C) $a \in [-\infty, 2] \setminus \{0\}$ (D) $a \in (0, \infty)$

Ans : (C)

Hint : By sandwich theorem

32. The differential equation of the family of curves y = e^x (A cos x + B sin x) where A, B are arbitrary constants is

(A) $\frac{d^2y}{dx^2} - 9x = 13$ (B) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ (C) $\frac{d^2y}{dx^2} + 3y = 4$ (D) $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - xy = 0$

Ans : (B)

Hint : y = e^x (A cos x + B sin x)

Differentiating w.r.t. x:-

$y' = y + e^x(-A\sin x + B\cos x)$

Differentiating w.r.t. x once again:-

$y'' = y' + (y' - y) + e^x(-A\cos x - B\sin x)$

$= 2y' - y - y \quad \Rightarrow y'' - 2y' + 2y = 0$

33. The equation $r\cos\left(\theta - \frac{\pi}{3}\right) = 2$ represents

(A) a circle (B) a parabola (C) an ellipse (D) a straight line

Ans : (D)

Hint : $r\cos\left(\theta-\frac{\pi}{3}\right) = 2 \quad \Rightarrow r\cos\theta \times \frac{1}{2} + r\sin\theta \times \frac{\sqrt{3}}{2} = 2 \Rightarrow x + \sqrt{3}y = 4$ (x = r cos θ, y = r sin θ)

$\therefore$ a straight line

34. The locus of the centre of the circles which touch both the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4ax$ externally is

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

Ans : (D)

Hint : Let, centre $\equiv$ (h, k) and radius = r for the variable circle

So, using $C_1C_2 = r_1 + r_2$ for both cases we have:

$h^2 + k^2 = (r + a)^2 \rightarrow (1)$ and $(h - 2a)^2 + k^2 = (r + 2a)^2 \rightarrow (2)$

Eq. (2) – Eq. (1), gives : $r = \dfrac{a-4h}{2} \rightarrow (3)$

Substitute (3) in (1) to get:

$12h^2 - 4k^2 - 24ah + 9a^2 = 0$

$\therefore$ locus : $12x^2 - 4y^2 - 24ax + 9a^2 = 0$ i.e. a hyperbola

35. Let each of the equations $x^2 + 2xy + ay^2 = 0$ & $ax^2 + 2xy + y^2 = 0$ represent two straight lines passing through the origin. If they have a common line, then the other two lines are given by

(A) $x - y = 0,\ x - 3y = 0$ (B) $x + 3y = 0,\ 3x + y = 0$ (C) $3x + y = 0,\ 3x - y = 0$ (D) $(3x - 2y) = 0,\ x + y = 0$

Ans : (B)

Hint : $\left(\dfrac{x}{y}\right)^2 + 2\left(\dfrac{x}{y}\right) + a = 0$ & $a\left(\dfrac{x}{y}\right)^2 + 2\left(\dfrac{x}{y}\right) + 1 = 0$ have exactly one root in common (taking $\dfrac{x}{y}$ as a single variable).

By, $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (a_1c_2 - a_2c_1)^2$

We get : $\Rightarrow$ a = 1 or –3

a cannot be 1

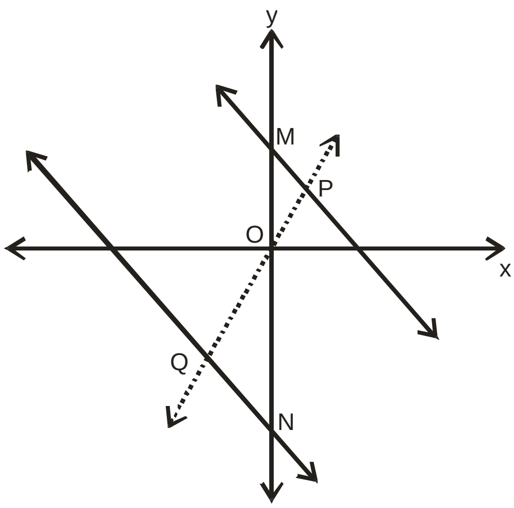
Taking a = –3, roots of 1st equation : 1, –3 and 2nd equation : 1, $-\dfrac{1}{3}$

So other lines : $\dfrac{x}{y} = -3$ and $\dfrac{x}{y} = -\dfrac{1}{3}$

36. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at P and Q respectively. The point O divides the segment PQ in the ratio

(A) 1 : 2 (B) 3 : 4 (C) 2 : 1 (D) 4 : 3

Ans : (B)



Hint : $\Delta OPM \sim \Delta OQN$ $\Rightarrow \dfrac{OP}{OQ} = \dfrac{OM}{ON} = \Rightarrow \dfrac{9/2}{12/2} = \dfrac{3}{4}$

37. Area in the first quadrant between the ellipses $x^2 + 2y^2 = a^2$ and $2x^2 + y^2 = a^2$ is

(A) $\frac{a^2}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$ (B) $\frac{3a^2}{4}\tan^{-1}\frac{1}{2}$ (C) $\frac{5a^2}{2}\sin^{-1}\frac{1}{2}$ (D) $\frac{9\pi a^2}{2}$

Ans : (A)

Hint : $A = \left(\frac{a^2}{6} + \int_{a/\sqrt{3}}^{a/\sqrt{2}} \sqrt{a^2 - 2x^2}\,dx\right) \times 2 = \frac{a^2}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

38. The equation of circle of radius $\sqrt{17}$ unit, with centre on the positive side of x-axis and through the point (0, 1) is

(A) $x^2 + y^2 - 8x - 1 = 0$ (B) $x^2 + y^2 + 8x - 1 = 0$ (C) $x^2 + y^2 - 9y + 1 = 0$ (D) $2x^2 + 2y^2 - 3x + 2y = 4$

Ans : (A)

Hint : Let centre be $\equiv (a, 0)$ $(a > 0)$

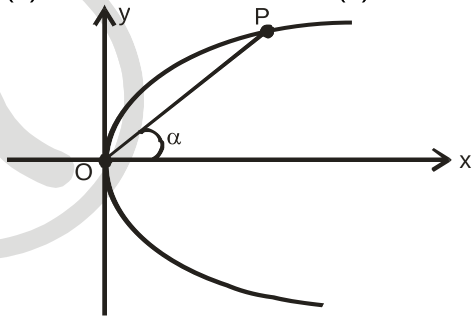
So, circle : $(x - a)^2 + y^2 = 17$; as it passes through (0, 1), so $a^2 + 1 = 17$ $\Rightarrow a = 4$ $(a \neq -4, a > 0)$

$\therefore$ Equation is : $x^2 + y^2 - 8x - 1 = 0$

39. The length of the chord of the parabola $y^2 = 4ax$ $(a > 0)$ which passes through the vertex and makes an acute angle α with the axis of the parabola is

(A) $\pm 4a \cot\alpha \operatorname{cosec}\alpha$ (B) $4a \cot\alpha \operatorname{cosec}\alpha$ (C) $-4a \cot\alpha \operatorname{cosec}\alpha$ (D) $4a \operatorname{cosec}^2\alpha$

Ans : (B)



Hint : Equation of OP:-

$y = x \tan\alpha$

Solving with $y^2 = 4ax$, we get :

$x^2 \tan^2\alpha = 4ax$ $\Rightarrow x = 4a\cot^2\alpha$

Substituting, $y = 4a\cot\alpha$

$\therefore P \equiv (4a\cot^2\alpha, 4a\cot\alpha)$

So, $\overline{OP} = \sqrt{16a^2\cot^4\alpha + 16a^2\cot^2\alpha}$ $= 4a\cot\alpha\operatorname{cosec}\alpha$ (as $0° < \alpha < 90°$, so $\cot\alpha > 0$, $\operatorname{cosec}\alpha > 0$)

40. A double ordinate PQ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is such that ΔOPQ is equilateral, O being the centre of the hyperbola. Then the eccentricity e satisfies the relation

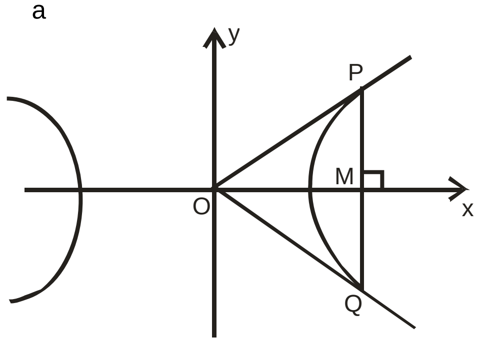
(A) $1 < e < \frac{2}{\sqrt{3}}$ (B) $e = \frac{2}{\sqrt{3}}$ (C) $e = \frac{\sqrt{3}}{2}$ (D) $e > \frac{2}{\sqrt{3}}$

Ans : (D)

Hint : Let, $P \equiv (a\sec\theta, b\tan\theta)$ $\angle POM = \tan 30° = \frac{1}{\sqrt{3}} = \frac{b}{a}\sin\theta$

$\Rightarrow \sin\theta = \frac{a}{b\sqrt{3}}$

$\Rightarrow \frac{b^2}{a^2} > \frac{1}{3}$ $\Rightarrow 1 + \frac{b^2}{a^2} > \frac{4}{3}$ $\Rightarrow e > \frac{2}{\sqrt{3}}$

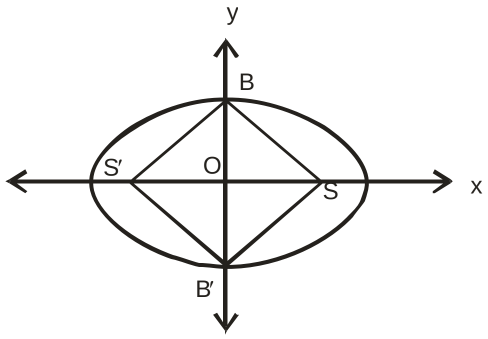


41. If B and B′ are the ends of minor axis and S and S′ are the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, then the area of the rhombus SBS′B′ will be

(A) 12 sq. unit (B) 48 sq. unit (C) 24 sq. unit (D) 36 sq. unit

Ans : (C)

Hint :



$B \equiv (0,3), B' \equiv (0,-3)$

$S \equiv (ae,0) \equiv \left(5\times\sqrt{1-\frac{9}{25}},0\right) \equiv (4,0)$ $\therefore \text{area}\,SBS'B' = 2\times\text{area}\Delta BSB' = 2\times\frac{1}{2}\times 4\times 6 = 24$ sq. unit

42. The equation of the latus rectum of a parabola is x + y = 8 and the equation of the tangent at the vertex is x + y = 12. Then the length of the latus rectum is

(A) $4\sqrt{2}$ unit (B) $2\sqrt{2}$ unit (C) 8 unit (D) $8\sqrt{2}$ unit

Ans : (D)

Hint : The distance between latus rectum and equation of tangent at vertex is 'a'. Here $a=\frac{4}{\sqrt{1+1}}=2\sqrt{2}$

So, length of latus rectum = 4a = $8\sqrt{2}$ unit

43. The equation of the plane through the point (2, –1, –3) and parallel to the lines $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-4}$ and $\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2}$ is

(A) 8x + 14y + 13z + 37 = 0 (B) 8x – 14y – 13z – 37 = 0

(C) 8x – 14y – 13z + 37 = 0 (D) 8x – 14y + 13z + 37 = 0

Ans : (*)

Hint : No correct options

44. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane 2x – 2y + z = 5 is

(A) $\frac{2\sqrt{3}}{5}$ (B) $\frac{\sqrt{2}}{10}$ (C) $\frac{4}{5\sqrt{2}}$ (D) $\frac{\sqrt{5}}{6}$

Ans : (B)

Hint : Let the angle be θ. Then, cos (90° – θ) = $\frac{\left(3\hat{i}+4\hat{j}+5\hat{k}\right).\left(2\hat{i}-2\hat{j}+\hat{k}\right)}{|3\hat{i}+4\hat{j}+5\hat{k}|\times|2\hat{i}-2\hat{j}+\hat{k}|} \Rightarrow \sin\theta=\frac{3}{\sqrt{50}\sqrt{9}}=\frac{\sqrt{2}}{10}$

45. Let f(x) = sin x + cos ax be periodic function. Then

(A) 'a' is any real number (B) 'a' is any irrational number

(C) 'a' is rational number (D) a = 0

Ans : (C)

Hint : LCM of rational multiple of same irrational is defined

46. The domain of $f(x) = \sqrt{\left(\frac{1}{\sqrt{x}} - \sqrt{x+1}\right)}$ is

(A) x > –1 (B) $(-1, \infty)\backslash\{0\}$ (C) $\left(0, \frac{\sqrt{5}-1}{2}\right]$ (D) $\left[\frac{1-\sqrt{5}}{2}, 0\right)$

Ans : (C)

Hint : $x^2 + x - 1 \le$ and x > 0

$x \in \left[\frac{-1-\sqrt{5}}{2}, \frac{\sqrt{5}-1}{2}\right]$

$x \in \left(0, \frac{\sqrt{5}-1}{2}\right]$

47. Let y = f(x) = $2x^2 - 3x + 2$. The differential of y when x changes from 2 to 1.99 is

(A) 0.01 (B) 0.18 (C) –0.05 (D) 0.07

Ans : (C)

Hint : $\Delta x = -0.01 \Rightarrow \Delta y = f'(x)\Delta x \Rightarrow \Delta y = f'(2)(-0.01) \Rightarrow \Delta y = -5 \times 0.01 \Rightarrow \Delta y = -0.05$

48. If $\lim_{x\to 0}\left(\frac{1+cx}{1-cx}\right)^{1/x} = 4$, then $\lim_{x\to 0}\left(\frac{1+2cx}{1-2cx}\right)^{1/x}$ is

(A) 2 (B) 4 (C) 16 (D) 64

Ans : (C)

Hint : $e^{2c} = 4 \Rightarrow e^{4c} = 16$

49. Let f : $\mathbb{R} \to \mathbb{R}$ be twice continuously differentiable (or f″ exists and is continuous) such that f(0) = f(1) = f′(0)=0. Then

(A) f″(c) = 0 for some $c \in \mathbb{R}$ (B) there is no point for which f″(x) = 0

(C) at all points f″(x) > 0 (D) at all points f″(x) < 0

Ans : (A)

Hint : f(0) = f(1) = 0

By Rolle's theorem $f'(c) = 0$ for some $c \in (0,1)$

Now, $f'(0) = f'(c) = 0$. Again by Rolle's theorem $f''(c_1) = 0$ $c_1 \in (0,c)$

50. Let f(x) = $x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 12$. Then

(A) f(x) has 13 non-zero real roots (B) f(x) has exactly one real root

(C) f(x) has exactly one pair of imaginary roots (D) f(x) has no real root

Ans : (B)

Hint : f′(x) > 0 $\forall x \in R$

f(x) = 0 has exactly one real root

CATEGORY - II (Q51 to Q65)

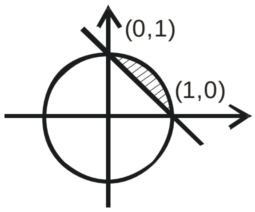
Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

51. The area of the region $\{(x,y): x^2 + y^2 \le 1 \le x + y\}$ is

(A) $\frac{\pi^2}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{4} - \frac{1}{2}$ (D) $\frac{\pi^2}{3}$

Ans : (C)

Hint :



Area of shaded region = $\frac{\pi}{4} - \frac{1}{2}$

52. In open interval $\left(0, \frac{\pi}{2}\right)$,

(A) cosx + xsinx < 1

(B) cosx + xsinx > 1

(C) no specific order relation can be ascertained between cosx + xsinx and 1

(D) cosx + xsinx < $\frac{1}{2}$

Ans : (B)

Hint : f(x) = cosx + xsinx – 1

$\Rightarrow$ f′(x) = –sinx + sinx + xcosx > 0; $x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow$ f(x) is increasing function

$\Rightarrow$ f(x) > f(0)

cosx + xsinx – 1 > 0

cosx + xsinx > 1

53. If the line y = x is a tangent to the parabola y = ax^2 + bx + c at the point (1, 1) and the curve passes through (–1, 0), then

(A) a = b = –1, c = 3 (B) a = b = $\frac{1}{2}$, c = 0 (C) a = c = $\frac{1}{4}$, b = $\frac{1}{2}$ (D) a = 0, b = c = $\frac{1}{2}$

Ans : (C)

Hint : $\frac{dy}{dx}$ = 2ax + b

at (1, 1) $\frac{dy}{dx}$ = 2a + b = 1 ...(1)

Now (1,1) and (–1, 0) satisfies the curve

a – b + c = 0 ...(2)

a + b + c = 1 ...(3)

$b = \frac{1}{2}$; $a = \frac{1}{4}$; $c = \frac{1}{4}$

54. If the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$, $\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$, and $\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is

(A) 1 (B) 0 (C) –1 (D) 2

Ans : (C)

Hint : $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}(1+abc) = 0$

$abc = -1$ [$\because \vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are non-coplanar vector]

55. Let z_1 and z_2 be two imaginary roots of $z^2 + pz + q = 0$, where p and q are real. The points z_1, z_2 and origin form an equilateral triangle if

(A) $p^2 > 3q$ (B) $p^2 < 3q$ (C) $p^2 = 3q$ (D) $p^2 = q$

Ans : (C)

Hint : $O^2 + z_1^2 + z_2^2 = z_1z_2$

$\Rightarrow z_1^2 + z_2^2 = z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$

$\Rightarrow p^2 = 3q$

56. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$ [a, b, c, d are all real], then P(x).Q(x) = 0 has

(A) at least two real roots (B) two real roots (C) four real roots (D) no real root

Ans : (A)

Hint : If $P(x) = ax^2 + bx + c$, $Q(x) = -ax^2 + dx + c$

$D_1 = b^2 - 4ac$

$D_2 = d^2 + 4ac$

$\Rightarrow D_1 + D_2 > 0$

Atleast two real roots.

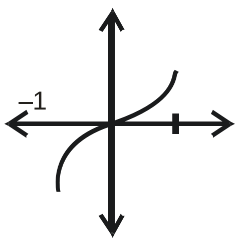
57. Let $A = \{x \in \mathbb{R} : -1 \le x \le 1\}$ & $f : A \to A$ be a mapping defined by $f(x) = x|x|$. Then f is

(A) injective but not surjective (B) surjective but not injective

(C) neither injective nor surjective (D) bijective

Ans : (D)

Hint : $f(x) = x|x| = \begin{cases} -x^2 & x \in [-1, 0) \\ x^2 & x \in [0,1] \end{cases}$



f(x) is bijective

58. Let f(x) = $\sqrt{x^2-3x+2}$ and g(x) = $\sqrt{x}$ be two given functions. If S be the domain of $f\circ g$ and T be the domain of $g\circ f$, then

(A) S = T (B) $S\cap T=\varphi$ (C) $S\cap T$ is a singleton (D) $S\cap T$ is an interval

Ans : (D)

Hint : f(x) = $\sqrt{(x-1)(x-2)}$, g(x) = $\sqrt{x}$

$$f(g(x))=\sqrt{\left(\sqrt{x}-1\right)\left(\sqrt{x}-2\right)}$$

$$S=\{x : x\in[0,1]\cup[4,\infty)\}$$

$$g(f(x))=\sqrt{\sqrt{(x-1)(x-2)}}$$

$$T=\{x : x\in(-\infty,1]\cup[2,\infty)\}$$

59. Let ρ_1 and ρ_2 be two equivalence relations defined on a non-void set S. Then

(A) both $\rho_1\cap\rho_2$ and $\rho_1\cup\rho_2$ are equivalence relations

(B) $\rho_1\cap\rho_2$ is equivalence relation but $\rho_1\cup\rho_2$ is not so.

(C) $\rho_1\cup\rho_2$ is equivalence relation but $\rho_1\cap\rho_2$ is not so

(D) neither $\rho_1\cap\rho_2$ nor $\rho_1\cup\rho_2$ is equivalence relation.

Ans : (B)

Hint : Union of two transitive may or may not be transitive

60. Consider the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The portion of the tangent at any point of the curve intercepted between the point of contact and the directrix subtends at the corresponding focus an angle of

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Ans : (C)

Hint : Property

61. A line cuts the x-axis at A (7, 0) and the y-axis at B (0, –5). A variable line PQ is drawn perpendicular to AB cutting the x-axis at P (a, 0) and the y-axis at Q (0, b). If AQ and BP intersect at R, the locus of R is

(A) $x^2+y^2+7x+5y=0$ (B) $x^2+y^2+7x-5y=0$ (C) $x^2+y^2-7x+5y=0$ (D) $x^2+y^2-7x-5y=0$

Ans : (C)

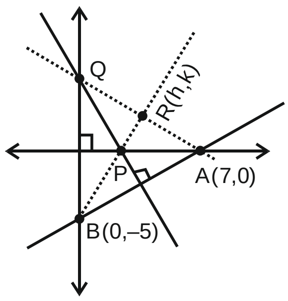
Hint :

P is orthocentre of ΔABQ

$m_{BR}\times m_{AR}=-1$

$$\Rightarrow\left(\frac{k+5}{h}\right)\times\left(\frac{k}{h-7}\right)=-1$$

$$\Rightarrow x^2+y^2-7x+5y=0$$



62. Let $0 < \alpha < \beta < 1$. Then $\lim_{n\to\infty}\sum_{k=1}^{n}\int_{1/(k+\beta)}^{1/(k+\alpha)}\frac{dx}{1+x}$ is

(A) $\log_e\frac{\beta}{\alpha}$ (B) $\log_e\frac{1+\beta}{1+\alpha}$ (C) $\log_e\frac{1+\alpha}{1+\beta}$ (D) ∞

Ans : (B)

Hint :

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left[\log|1+x|\right]_{\frac{1}{k+\beta}}^{\frac{1}{k+\alpha}}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(\log\left(1+\frac{1}{k+\alpha}\right)-\log\left(1+\frac{1}{k+\beta}\right)\right)$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\left(\log\left(\frac{k+\alpha+1}{k+\alpha}\right)-\log\left(\frac{k+\beta+1}{k+\beta}\right)\right)$$

$$=\log\left(\frac{\beta+1}{\alpha+1}\right)$$

63. $\lim_{x\to1}\left(\frac{1}{\ln x}-\frac{1}{(x-1)}\right)$

(A) Does not exist (B) 1 (C) $\frac{1}{2}$ (D) 0

Ans : (C)

Hint : $\underset{x\to1}{\text{lt}}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=\underset{x\to1}{\text{lt}}\frac{(x-1)-\ln x}{(x-1)\ln x}=\frac{1}{2}$

Using L.H. rule twice

64. Let $y=\frac{1}{1+x+\ln x}$, Then

(A) $x\frac{dy}{dx}+y=x$ (B) $x\frac{dy}{dx}=y(y\ln x-1)$ (C) $x^2\frac{dy}{dx}=y^2+1-x^2$ (D) $x\left(\frac{dy}{dx}\right)^2=y-x$

Ans : (B)

Hint : $x\frac{dy}{dx}=y(y\ln x-1)$

65. Consider the curve $y = be^{-x/a}$ where a and b are non-zero real numbers. Then

(A) $\frac{x}{a}+\frac{y}{b}=1$ is tangent to the curve at (0, 0)

(B) $\frac{x}{a}+\frac{y}{b}=1$ is tangent to the curve where the curve crosses the axis of y

(C) $\frac{x}{a}+\frac{Y}{b}=1$ is tangent to the curve at (a, 0)

(D) $\frac{x}{a}+\frac{y}{b}=1$ is tangent to the curve at (2a, 0)

Ans : (B)

Hint : $y-b=-\frac{b}{a}(x) \Rightarrow \frac{x}{a}+\frac{y}{b}=1$

CATEGORY - III (Q66 to Q75)

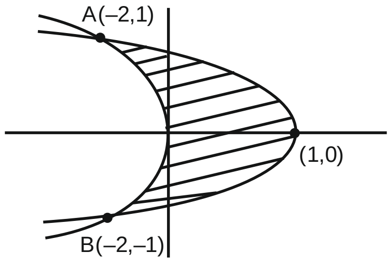
Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. The area of the figure bounded by the parabola $x=-2y^2$, $x=1-3y^2$ is

(A) $\frac{1}{3}$ square unit (B) $\frac{4}{3}$ square unit (C) 1 square unit (D) 2 square unit

Ans : (B)

Hint :



$$A=\int_{-1}^{1}\left(1-3y^2\right)-\left(-2y^2\right)dy$$

$$\Rightarrow A=2\int_{0}^{1}\left(1-y^2\right)dy=\frac{4}{3}$$

67. A particle is projected vertically upwards. If it has to stay above the ground for 12 seconds, then

(A) velocity of projection is 192 ft/sec
(B) greatest height attained is 600 ft
(C) velocity of projection is 196 ft / sec
(D) greatest height attained is 576 ft

Ans : (A,D)

Hint : V = u – gt at t = 6

u – gt = 0

$\Rightarrow u=6g=192\text{ft/sec}$ $\left(g=32\text{ft/sec}^2\right)$ ....(i)

$$x=ut=\frac{1}{2}gt^2$$

$$=192.6-\frac{1}{2}32.6^2$$

$= 576\text{ft}$

68. The equation $x^{(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5} = 3\sqrt{3}$ has

(A) at least one real root (B) exactly one real root

(C) exactly one irrational root (D) complex roots

Ans : (A,C)

Hint : $2t^3 - 9t^2 + 10t - 3 = 0,\ t = \log_3 x$

$\Rightarrow (t-3)(t-1)\left(t-\frac{1}{2}\right) = 0$

$\Rightarrow x = 3^3, 3^1, 3^{1/2}$

69. In a certain test, there are n questions. In this test 2^{n-i} students gave wrong answers to at least i questions, where i = 1, 2, ... n. If the total number of wrong answers given is 2047, then n is equal to

(A) 10 (B) 11 (C) 12 (D) 13

Ans : (B)

Hint : Total students was gave wrong answer to exactly i – questions = $2^{n-i} - 2^{n-(i+1)}$

Total wrong answer given = $\sum i \times \left(2^{n-i} - 2^{n-(i+1)}\right)$

$\Rightarrow 2^{n-1} + ...1 = 2047$

$\Rightarrow 2^n = 2048$

$\Rightarrow n = 11$

70. A and B are independent events. The probability that both A and B occur is $\frac{1}{20}$ and the probability that neither of them occurs is $\frac{3}{5}$. The probability of occurrence of A is

(A) $\frac{1}{2}$ (B) $\frac{1}{10}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

Ans : (C,D)

Hint : $P(A' \cap B') = \frac{3}{5}$

$\Rightarrow 1 - P(A \cup B) = \frac{3}{5}$

$\Rightarrow P(A \cup B) = \frac{2}{5}$

$\Rightarrow P(A) + P(B) - P(A).P(B) = \frac{2}{5}$

$\Rightarrow P(A) + P(B) = \frac{9}{20}$ and $P(A).P(B) = \frac{1}{20}$

71. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is

(A) $\frac{x}{2} - \frac{y}{3} = 1$ (B) $\frac{x}{-2} + \frac{y}{1} = 1$ (C) $-\frac{x}{3} + \frac{y}{2} = 1$ (D) $\frac{x}{1} - \frac{y}{2} = 1$

Ans : (A,B)

Hint : Let line be $\frac{x}{a}+\frac{y}{b}=1$, then a+b = –1

$$\frac{4}{a}-\frac{3}{1+a}=1 \Rightarrow a = \pm 2$$

72. Consider a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ at any point. The locus of the midpoint of the portion intercepted between the axes is

(A) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (B) $\frac{x^2}{4}+\frac{y^2}{2}=1$ (C) $\frac{1}{3x^2}+\frac{1}{4y^2}=1$ (D) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

Ans : (D)

Hint : Tangent at $P(x_1, y_1)$

$$\frac{xx_1}{2}+\frac{yy_1}{1}=1$$

Let mid point of intercept be P(h, k)

$$h=\frac{1}{x_1}, k=\frac{1}{2y_1} \text{ or } x_1=\frac{1}{h}, y_1=\frac{1}{2k}$$

locus is $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

73. Let $y=\frac{x^2}{(x+1)^2(x+2)}$. Then $\frac{d^2y}{dx^2}$ is

(A) $2\left[\frac{3}{(x+1)^4}-\frac{3}{(x+1)^3}+\frac{4}{(x+2)^3}\right]$ (B) $3\left[\frac{2}{(x+1)^3}+\frac{4}{(x+1)^2}-\frac{5}{(x+2)^3}\right]$

(C) $\frac{6}{(x+1)^3}-\frac{4}{(x+1)^2}+\frac{3}{(x+1)^3}$ (D) $\frac{7}{(x+1)^3}-\frac{3}{(x+1)^2}+\frac{2}{(x+1)^3}$

Ans : (A)

Hint : By partial fraction technique

$$y=\frac{x^2}{(x+1)^2(x+2)}=\frac{4}{(x+2)}-\frac{3}{(x+1)}+\frac{1}{(x+1)^2}$$

$$\Rightarrow y''=\frac{6}{(x+1)^4}-\frac{6}{(x+1)^3}+\frac{8}{(x+2)^3}$$

74. Let f(x)= $\frac{1}{3}x\sin x-(1-\cos x)$. The smallest positive interger k such that $\lim_{x\to 0}\frac{f(x)}{x^k}\neq 0$ is

(A) 4 (B) 3 (C) 2 (D) 1

Ans : (C)

Hint : $\underset{x\to 0}{\text{lt}}\frac{x\sin x-3(1-\cos x)}{3x^k}=\frac{1}{3}\underset{x\to 0}{\text{lt}}\left(\frac{\sin x/2}{x/2}\right)\underset{x\to 0}{\text{lt}}\left(\frac{2x\cos x/2-6\sin x/2}{2x^{k-1}}\right)$

$k-1=1 \Rightarrow k=2$

75. Tangent is drawn at any point P (x, y) on a curve, which passes through (1, 1). The tangent cuts X-axis and Y-axis at A and B respectively. If AP:BP = 3:1, then

(A) the differential equation of the curve is $3x\frac{dy}{dx}+y=0$

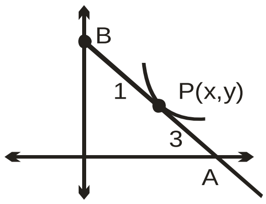
(B) the differential equation of the curve is $3x\frac{dy}{dx}-y=0$

(C) the curve passes through $\left(\frac{1}{8},2\right)$

(D) the normal at (1, 1) is x+3y = 4

Ans : (A,C)

Hint :



$\frac{PA}{PB}=\frac{3}{1}$

Equation of tangent AB is $Y-y=\frac{dy}{dx}(X-x)$

$A\left(\frac{xy'-y}{y'},0\right)$ and B(0, y–xy′)

Using section formula : $y=\frac{1\times 0+3\times(y-xy')}{4}$ $\Rightarrow 4y=3y-3xy' \Rightarrow 3xy'=-y \Rightarrow \frac{3xdy}{dx}+y=0$

$\frac{3dy}{y}+\frac{dx}{x}=0$

$xy^3=1$

$\frac{dy}{dx_{(1,1)}}=\frac{-1}{3}$

Slope of Normal = 3

Equation of Normal $\Rightarrow y-1=3(x-1) \Rightarrow \boxed{y-3x+2=0}$

❑ ❑ ❑

WBJEE - 2020

Answer Keys by

Aakash Institute, Kolkata Centre

PHYSICS & CHEMISTRY

Q.No.	✿	✚	◯	◻
01	B	C	C	C
02	D	B	C	B
03	B	A	B	C
04	C	D	A	A
05	A	A	A	B
06	D	D	B	D
07	A	C	D	A
08	D	C	B	C
09	B	B	C	B
10	A	A	A	A
11	A	A	D	D
12	A	B	A	A
13	C	D	D	D
14	B	B	B	C
15	C	C	A	C
16	A	A	A	B
17	B	D	A	A
18	D	A	C	A
19	A	D	B	B
20	C	B	C	D
21	B	A	A	B
22	A	A	B	C
23	D	A	D	A
24	A	C	A	D
25	D	B	C	A
26	C	C	B	D
27	C	A	A	B
28	B	B	D	A
29	A	D	A	A
30	A	A	D	A
31	C	C	B	A
32	B	B	C	C
33	A	C	B	B
34	C	B	A	C
35	B	A	C	B
36	B	D	A,C,D	A
37	D	A,C,D	B	D
38	A	B	D	A,C,D
39	D	D	A	B
40	A,C,D	A	D	D
41	C	B	A	A
42	C	B	C	B
43	B	C	D	B
44	D	C	C	D
45	C	A	C	A
46	A	C	B	C
47	D	D	D	C
48	B	C	C	A
49	A	C	A	D
50	A	B	D	A
51	B	D	B	C
52	B	C	A	A
53	D	A	A	D
54	A	D	B	D
55	C	B	B	B
56	C	A	D	B
57	A	A	A	C
58	D	B	C	C
59	A	B	C	A
60	C	D	A	C
61	A	A	D	D
62	D	C	A	C
63	D	C	C	C
64	B	A	A	B
65	B	D	D	D
66	C	A	D	C
67	C	C	B	A
68	A	A	B	D
69	C	D	C	B
70	D	D	C	A
71	D	C	D	D
72	B	D	D	C
73	D	D	B	D
74	C	B	D	D
75	D	D	C	B
76	A,B	A,C,D	A,C	A,B,D
77	A,B,D	A,C	A,B	A,C,D
78	A,B,D	A,B	A,B,D	A,C
79	A,C,D	A,B,D	A,B,D	A,B
80	A,C	A,B,D	A,C,D	A,B,D

Code - ✿

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ANSWERS & HINT
for
WBJEE - 2020
SUB : PHYSICS & CHEMISTRY

PHYSICS

CATEGORY - I (Q1 to Q30)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

1. The intensity of light emerging from one of the slits in a Young's double slit experiment is found to be 1.5 limes the intensity of light emerging from the other slit. What will be the approximate ratio of intensity of an interference maximum to that of an interference

   (A) 2.25 (B) 98 (C) 5 (D) 9.9

   Ans : (B)

   Hint : $I_1 = 1.5 I_2$

   $\frac{I_1}{I_2} = \frac{3}{2}$. $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)^2 = 98$

2. In a Frounhofer diffraction experiment, a single slit of width 0.5 mm is illuminated by a monochromatic light of wavelength 600 nm. The diffraction pattern is observed on a screen at a distance of 50 cm from the slit. What will be the linear separation of the first order minima ?

   (A) 1.0 nm (B) 1.1 nm (C) 0.6 mm (D) 1.2 nm

   Ans : (D)

   Hint : d=0.5 mm.

   λ=600 nm. Width of central maxima = $\frac{2\lambda D}{d}$

   D = 50 cm

   $\frac{2\lambda D}{d} = \frac{2\times 600\times 50\times 10^{-2}\times 10^{-9}}{0.5\times 10^{-3}} = 1.2\text{mm}$

3. If R is the Rydberg Constant in cm^{-1}, then hydrogen atom does not emit any radiation of wave-length in the range of

   (A) $\frac{1}{R}$ to $\frac{4}{3R}$ cm (B) $\frac{7}{5R}$ to $\frac{19}{5R}$ cm (C) $\frac{4}{R}$ to $\frac{36}{5R}$ cm (D) $\frac{9}{R}$ to $\frac{144}{7R}$ cm

Ans : (B)

Hint : $\frac{1}{\lambda}=R\left[\frac{1}{n_{f^2}}-\frac{1}{n_{i^2}}\right]$

For range of wavelengths:-

n_i = 1, 2, 3, ... for Lyman, Balmer, Paschen, ...

$n_f = n_i + 1$ and $n_f = \infty$ for upper and lower range

Thus, Lyman : $\left[\frac{1}{R}\text{to}\frac{4}{3R}\right]$, Balmer : $\left[\frac{4}{R}\text{to}\frac{36}{5R}\right]$, Paschen : $\left[\frac{9}{R}\text{to}\frac{144}{36R}\right]$, Bracket : $\left[\frac{16}{R}\text{to}\frac{400}{9R}\right]$, Pfund : $\left[\frac{25}{R}\text{to}\frac{900}{11R}\right]$

4. A nucleus X emits a beta particle to produce a nucleus Y. If their atomic masses are M_x and M_y respectively, the maximum energy of the beta particle emitted is (where m_e is the mass of an electron and c is the velocity of light)

(A) $(M_x - M_y - m_e)c^2$ (B) $(M_x - M_y + m_e)c^2$ (C) $(M_x - M_y)c^2$ (D) $(M_x - M_y - 2m_e)c^2$

Ans : (C)

Hint : ${}_zX^A \rightarrow \underset{z+1}{\overset{A}{Y}} + \underset{-1}{e^0} + \bar{\nu} + Q$

$$m_x = M_x - zm_e$$
$$m_y = M_y - (z+1)m_e$$
$$m_x - m_y = M_x - M_y + m_e$$

$E = \Delta mC^2 = (m_x - m_y - m_e)C^2 = (M_x - M_y)C^2$

5. For nuclei with mass number close to 119 and 238, the binding energies per nucleon are approximately 7.6 MeV and 8.6 MeV respectively. If a nucleus of mass number 238 breaks into two nuclei of nearly equal masses, what will be the approximate amount of energy released in the process of fission ?

(A) 214 MeV (B) 119 MeV (C) 2047 MeV (D) 1142 MeV

Ans : (A)

Hint : $119 \rightarrow 7.6\text{MeV}$

$238 \rightarrow 8.6\text{MeV}$

$238 \rightarrow 119 + 119$

$E_i = 238 \times 8.6$

$E_f = 238 \times 7.6$

$\therefore E_f - E_i = -238\text{MeV}$

A negative value indicates the fission cannot take place. The data given are incorrect. The BE per nucleon values are swapped in question, it should have been 7.6 MeV for 238 and 8.6 MeV for 119. Thus Δ E=+238MeV. Thus 214 is closest and will be taken as right answer

6. A common emitter transistor amplifier is connected with a load resistance of 6 kΩ. When a small a.c. signal of 15 mV is added to the base emitter voltage, the alternating base current is 20 μA and the alternating collector current is 1.8 mA. What is the voltage gain of the amplifier ?

(A) 90 (B) 640 (C) 900 (D) 720

Ans : (D)

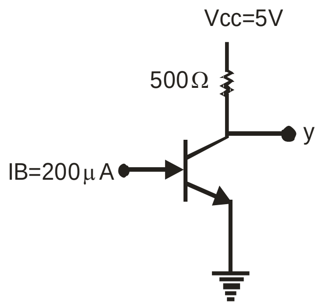
Hint : $R_C = 6 \times 10^3\ \Omega$

$\Delta I_B R_B = 15\times10^{-3}V$

$\Delta I_B = 20\times10^{-6}A$

$\Delta I_C = 1.8\times10^{-3}A$

$$A_V = \beta\times\frac{R_C}{R_B} = \frac{\Delta I_C}{\Delta I_B}\times\frac{R_C}{R_B} = \frac{1.8\times10^{-3}}{20\times10^{-6}}\times\frac{6\times10^{3}\times20\times10^{-6}}{15\times10^{-3}} = 720$$

7. 

In the circuit shown, the value of β of the transistor is 48. If the base current supplied is 200 μA, what is the voltage at the terminal Y ?

(A) 0.2V (B) 0.5V (C) 4V (D) 4.8V

Ans : (A)

Hint : $\beta = 48$

$I_B = 200\mu A$

$I_C = \beta I_B = 48\times200\times10^{-6}$

$V_{CC} = I_C R_C + V_{CE}$ $\therefore V_{CE} = 5-(96\times10^{-4})\times500 = 5-4.8 = 0.2\,\text{volt}$

8. The frequency ν of the radiation emitted by an atom when an electron jumps from one orbit to another is given by $\nu = k\,\delta E$, where k is a constant and δE is the change in energy level due to the transition. Then dimension of k is

(A) ML^2T^{-2} (B) the same dimension of angular momentum
(C) ML^2T^{-1} (D) $M^{-1}L^{-2}T$

Ans : (D)

Hint : $\nu = k\delta E$

$$[k] = \left[\frac{\nu}{\delta E}\right] = \frac{[T^{-1}]}{[ML^2T^{-2}]}$$

$[k] = [M^{-1}L^{-2}T^{1}]$

9. Consider the vectors $\vec{A} = \hat{i}+\hat{j}-\hat{k}$, $\vec{B} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{C} = \frac{1}{\sqrt{5}}(\hat{i}-2\hat{j}+2\hat{k})$. What is the value of $\vec{C}.(\vec{A}\times\vec{B})$?

(A) 1 (B) 0 (C) $3\sqrt{2}$ (D) $18\sqrt{5}$

Ans : (B)

Hint : $\vec{C}.\left(\vec{A}\times\vec{B}\right)=\begin{vmatrix}1 & 1 & -1\\ 2 & -1 & 1\\ 1 & -2 & 2\end{vmatrix}=1(-2+2)-1(4-1)-1(-4+1)=-3+3=0$

10. A fighter plane, flying horizontally with a speed 360 kmph at an altitude of 500 m drops a bomb for a target straight ahead of it on the ground. The bomb .should be dropped at what approximate distance ahead of the target ? Assume that acceleration due to gravity (g) is 10 ms^{-2}. Also neglect air drag

(A) 1000 m (B) $50\sqrt{5}$ m (C) $500\sqrt{5}$ m (D) 866 m

Ans : (A)

Hint : v = 360 km/h = 100 m/s.

h=500 m

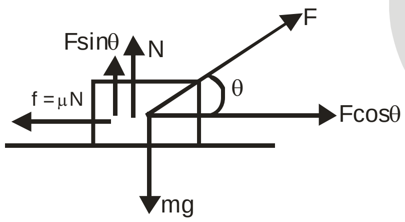
$R = u\sqrt{\frac{2h}{g}}=100\times\sqrt{\frac{2\times500}{10}}=1000m$

11. A block of mass m rests on a horizontal table with a co-efficient of static friction μ. What minimum force must be applied on the block to drag it on the table ?

(A) $\frac{\mu}{\sqrt{1+\mu^2}}mg$ (B) $\frac{\mu-1}{\mu+1}mg$ (C) $\frac{\mu}{\sqrt{1-\mu^2}}mg$ (D) μmg

Ans : (C)

Hint :



F sinθ + N = mg N = mg – F sinθ

F cosθ = μ(mg – F sinθ) $F=\frac{\mu mg}{\cos\theta+\mu\sin\theta}$,

for F_{min}, $\frac{d}{d\theta}[\cos\theta+\mu\sin\theta]=0$

$\therefore$ tan θ = μ

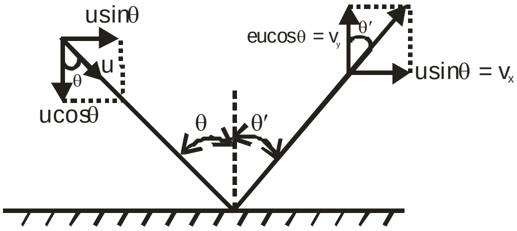
$\therefore$ $\boxed{F_{min}=\frac{\mu mg}{\sqrt{1+\mu^2}}}$

12. A tennis ball hits the floor with a speed v at an angle θ with the normal to the floor. If the collision is inelastic and the co-efficient of restitution is ε, what will be the angle of reflection ?

(A) $\tan^{-1}\left(\frac{\tan\theta}{\varepsilon}\right)$ (B) $\sin^{-1}\left(\frac{\sin\theta}{\varepsilon}\right)$ (C) $\theta\varepsilon$ (D) $\theta\frac{2\varepsilon}{\varepsilon+1}$

Ans : (A)

Hint :



$\tan\theta' = \dfrac{u\sin\theta}{eu\cos\theta}$ $\boxed{\tan\theta' = \dfrac{1}{e}\tan\theta}$

Since the floor is smooth. Hence tangential component of velocity remains unchanged.

13. The bob of a swinging seconds pendulum (one whose time period is 2 s) has a small speed v_0 at its lowest point. Its height from this lowest point 2.25 s after passing through it is given by

(A) $\dfrac{v_0^2}{2g}$ (B) $\dfrac{v_0^2}{g}$ (C) $\dfrac{v_0^2}{4g}$ (D) $\dfrac{9v_0^2}{4g}$

Ans : (C)

Hint : T = 2 sec. at t = T + T/8

t = T + T/8 $v = A\omega\cos\omega t = v_0/\sqrt{2}$

By Mechanical energy conservation:

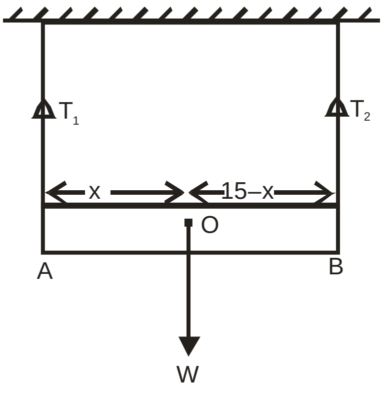
$\left(\dfrac{v_0}{\sqrt{2}}\right)^2 = v_0^2 - 2gh$ $\boxed{h = \dfrac{v_0^2}{4g}}$

14. A steel and a brass wire, each of length 50 cm and cross-sectional area 0.005 cm^2 hang from a ceiling and are 15 cm apart. Lower ends of the wires are attached to a light horizontal bar. A suitable downward load is applied to the bar so that each of the wires extends in length by 0.1 cm. At what distance from the steel wire the load must be applied ?
[Young's modulus of steel is 2×10^{12} dynes/cm^2 and that of brass is 1×10^{12} dynes/cm^2]

(A) 7.5 cm (B) 5 cm (C) 10 cm (D) 3 cm

Ans : (B)

Hint :

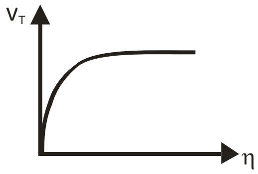


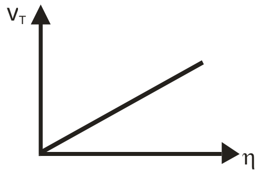
At equilibrium, Taking torque about point O

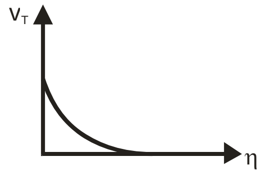
$T_1x = T_2(15 - x) \Rightarrow Y_1x = Y_2(15 - x)$

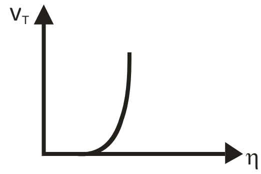
$\left[Y = \dfrac{T}{A} \times \dfrac{L}{\ell};\ T \propto Y\right]$ $1 \times 10^{12}x = 2 \times 10^{12}(15 - x)$; $3x = 15$, So, x = 5 cm

15. Which of the following diagrams correctly shows the relation between the terminal velocity v_T of a spherical body falling in a liquid and viscosity η of the liquid ?

(A) 

(B) 

(C) 

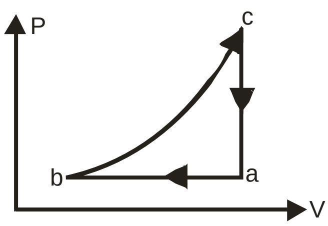
(D) 

Ans : (C)

Hint : At terminal speed.

$V_T = \frac{2r^2}{9\eta}(\rho_S - \rho_L)g$ (Assuming other factor constant)

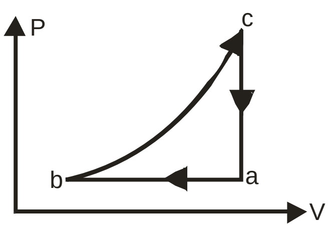
So, $V_T \propto \frac{1}{\eta}$ (rectangular hyperbola)

16. 

An ideal gas undergoes the cyclic process *abca* as shown in the given P-V diagram. It rejects 50 J of heat during *ab* and absorbs 80 J of heat during *ca*. During *bc*, there is no transfer of heat and 40 J of work is done by the gas. What should be the area of the closed curve *abca* ?

(A) 30 J (B) 40 J (C) 10 J (D) 90 J

Ans : (A)

Hint : 

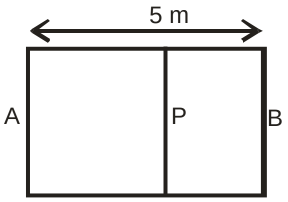
Area of cycle = W_{total}

$= \Delta Q_{net} = \Delta Q_{AB} + \Delta Q_{BC} + \Delta Q_{CA} = -50 + 0 + 80 = 30$ J

NOTE: In process BC, $W_{BC} > 0$ and $\Delta U_{BC} > 0$

$\therefore \Delta Q_{BC} = \Delta U_{BC} + W_{BC} > 0$

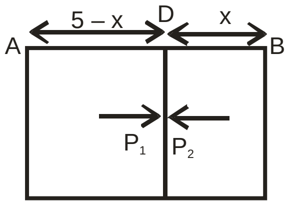
Thus $\Delta Q_{BC} = 0$ as given in question isn't possible. But we have neglected this technical mistake while answering.

17. 

A container AB in the shape of a rectangular parallelopiped of length 5 m is divided internally by a movable partition P as shown in the figure. The left compartment is filled with a given mass of an ideal gas of molar mass 32 while the right compartment is filled with an equal mass of another ideal gas of molar mass 18 at same temperature. What will be the distance of P from the left wall A when equilibrium is established ?

(A) 2.5 m (B) 1.8 m (C) 3.2 m (D) 2.1 m

Ans : (B)

Hint : 

At equilibrium $P_1 = P_2$, $T_1 = T_2$ (assuming wall to be conducting)

$$\frac{P_1V_1}{\mu_1} = \frac{P_1V_1}{\mu_2} = RT$$

$$\frac{(5-x)32}{m} = \frac{x \times 18}{m}$$ So, x = 3.2 m

Hence the distance of the piston from A = 5 – 3.2 = 1.8 m

18. When 100 g of boiling water at 100°C is added into a calorimeter containing 300 g of cold water at 10°C, temperature of the mixture becomes 20°C. Then a metallic block of mass 1 kg at 10°C is dipped into the mixture in the calorimeter. After reaching thermal equilibrium, the final temperature becomes 19°C. What is the specific heat of the metal in C.G.S. unit ?

(A) 0.01 (B) 0.3 (C) 0.09 (D) 0.1

Ans : (D)

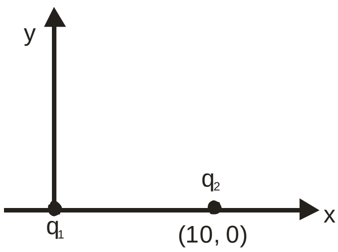
Hint : Let, heat capacity of calorimeter = ms

$100 \times 1 \times 80 = 300 \times 1 \times 10 + ms \times 10$

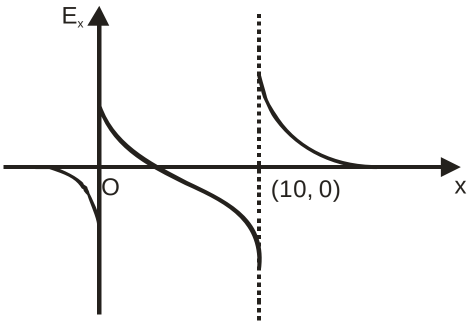
$\therefore$ ms = 500 Cal/°C

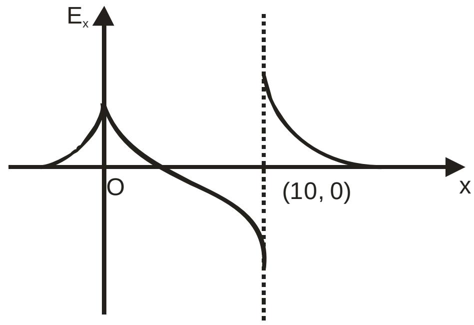
Again, $(100 + 300) \times 1 \times 1 + 500 \times 1 = 1000 \times S_b \times 9$

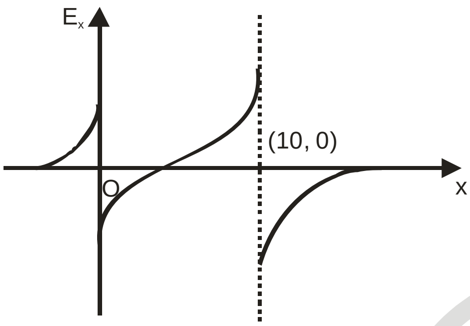
$\therefore$ $S_b = 0.1$ Cal/g°C

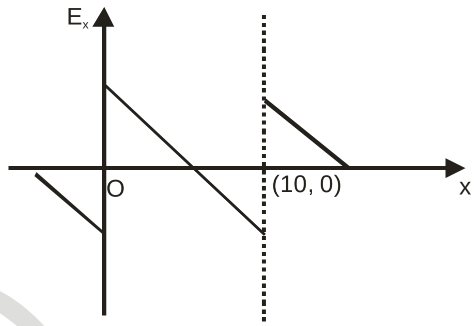
19. 

As shwon in the figure, a point charge $q_1 = +1 \times 10^{-6}$ C is placed at the origin in x-y plane and another point charge $q_2 = +3 \times 10^{-6}$ C is placed at the co-ordinate (10, 0). In that case, which of the following graph(s) shows most correctly the electric field vector in E_x in x-direction ?

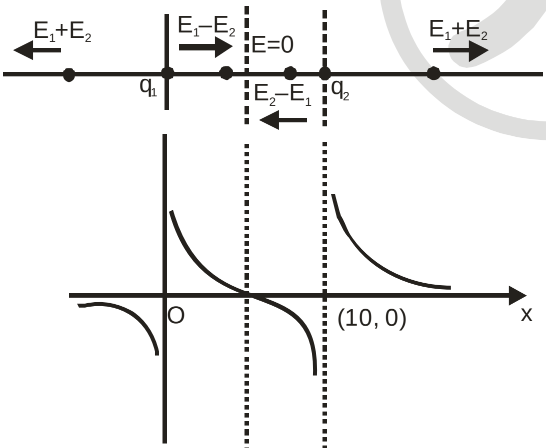
(A)


(B)


(C)


(D)


Ans : (A)

Hint :


20. Four identical point masses, each of mass m and carrying charge +q are placed at the corners of a square of sides 'a' on a frictionless plain surface. If the particles are released simultaneously, the kinetic energy of the system when they are infinitely far apart is

(A) $\frac{q^2}{a}\left(2\sqrt{2}+1\right)$ (B) $\frac{q^2}{a}\left(\sqrt{2}+2\right)$ (C) $\frac{q^2}{a}\left(\sqrt{2}+4\right)$ (D) $\frac{q^2}{a}\left(\sqrt{2}+1\right)$

Ans : (C)

Hint : $KE_i + PE_i = KE_f + PE_f$

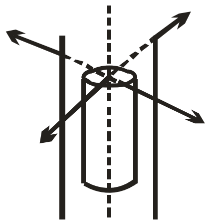
$\therefore KE_f = PE_i = 4\frac{Kq^2}{a} + 2\frac{Kq^2}{\sqrt{2}a} = \frac{Kq^2}{a}\left(4+\sqrt{2}\right)$, K = 1 in C.G.S. unit

21. A very long charged solid cylinder of radius 'a' contains a uniform charge density ρ. Dielectric constant of the material of the cylinder is k. What will be the magnitude of electric field at a radial distance 'x' (x < a) from the axis of the cylinder ?

(A) $\rho\frac{x}{\varepsilon_0}$ (B) $\rho\frac{x}{2k\varepsilon_0}$ (C) $\rho\frac{x^2}{2a\varepsilon_0}$ (D) $\rho\frac{x^2}{2k}$

Ans : (B)

Hint : Using Gauss's Law



$$E(2\pi x\ell) = \frac{\rho(\pi x^2\ell)}{k\varepsilon_0} \quad \therefore E = \frac{\rho x}{2k\varepsilon_0}$$

22. A galvanometer can be converted to a voltmeter of full-scale deflection V_0 by connecting a series resistance R_1 and can be converted to an ammeter of full-scale deflection I_0 by connecting a shunt resistance R_2. What is the current flowing through the galvanometer at its full-scale deflection ?

(A) $\frac{V_0 - I_0R_2}{R_1 - R_2}$ (B) $\frac{V_0 + I_0R_2}{R_1 + R_2}$ (C) $\frac{V_0 - I_0R_2}{R_2 - R_1}$ (D) $\frac{V_0 + I_0R_1}{R_1 + R_2}$

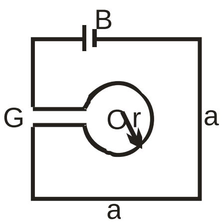
Ans : (A)

Hint : $R_1 = \frac{V_0}{I_g} - G$..........(1)

$R_2 = \frac{I_gG}{I_0 - I_g}$............(2)

Eliminating G we get

$$I_g = \frac{V_0 - I_0R_2}{R_1 - R_2}$$

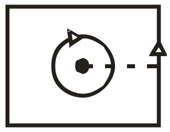
23. 

As shown in the figure, a single conducting wire is bent to form a loop in the form of a circle of radius 'r' concentrically inside a square of side 'a', where a : r = 8 : π. A battery B drives a current through the wire. If the battery B and the gap G are of negligible sizes, determine the strength of magnetic field at the common centre O.

(A) $\frac{\mu_0 I}{2\pi a}\sqrt{2}(\sqrt{2}-1)$ (B) $\frac{\mu_0 I}{2\pi a}(\sqrt{2}+1)$ (C) $\frac{\mu_0 I}{\pi a}2\sqrt{2}(\sqrt{2}+1)$ (D) $\frac{\mu_0 I}{\pi a}2\sqrt{2}(\sqrt{2}-1)$

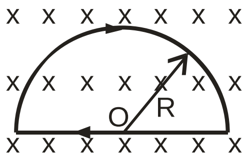
Ans : (D)

Hint : $\frac{a}{r} = \frac{8}{\pi}$, $r = \frac{\pi a}{8}$



$$B = \frac{\mu_0 I}{2r} - \frac{4\mu_0 I}{4\pi a/2} \times \sqrt{2} = \frac{\mu_0 I}{2r} - \frac{2\sqrt{2}\mu_0 I}{\pi a} = \frac{\mu_0 I \times 8}{2\pi a} - \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

$$= \frac{\mu_0 I}{\pi a}\left(4 - 2\sqrt{2}\right) = \frac{\mu_0 I}{\pi a} 2\sqrt{2}\left(\sqrt{2} - 1\right)$$

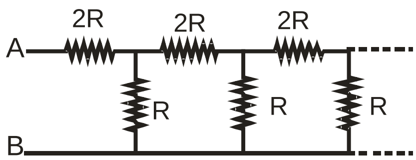
24. 

As shown in the figure, a wire is bent to form a D-shaped loop, carrying current **I**, where the curved part is semi-circle of radius R. the loop is placed in a uniform magnetic field $\vec{B}$, which is directed into the plane of the paper. The magnetic force left by the closed loop is

(A) 0 (B) IRB (C) 2IRB (D) $\frac{1}{2}$IRB

Ans : (A)

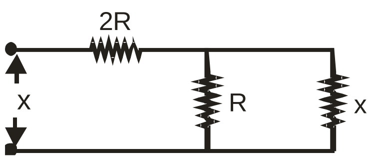
Hint : Zero (Net magnetic force on a closed current loop in a uniform $\vec{B}$ is zero)

25. 

What will be the equivalent resistance between the terminals A and B of the infinite resistive network shown in the figure ?

(A) $\frac{\left(\sqrt{3}+1\right)R}{2}$ (B) $\frac{\left(\sqrt{3}-1\right)R}{2}$ (C) $3\frac{R}{2}$ (D) $\left(\sqrt{3}+1\right)R$

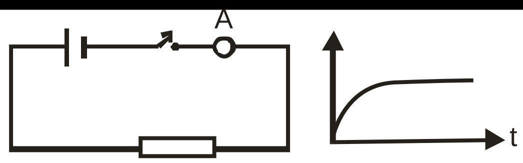
Ans : (D)

Hint : 

$$x = 2R + \frac{Rx}{R+x}$$

$x^2 - 2Rx - 2R^2 = 0$

$$x = \left(1 + \sqrt{3}\right)R$$

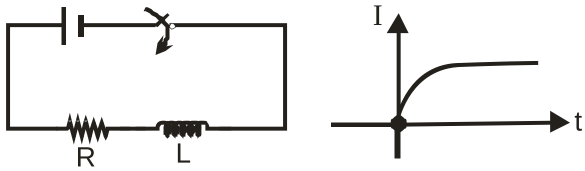
26. 

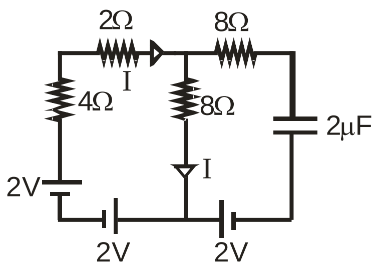
When a DC voltage is applied at the two ends of a circuit kept in closed box, it is observed that the current gradually increases from zero to a certain value and then remains constant. What do you think that the circuit contains ?

(A) A resistor alone
(B) A capacitor alone
(C) A resistor and an inductor in series
(D) A resistor and a capacitor in series

Ans : (C)

Hint : LR circuit transient

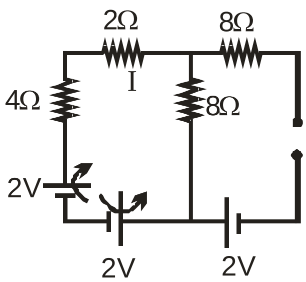


27. 

Consider the circuit shown. If all the cells have negligible internal resistance, what will be the current through the 2Ω resistor when steady state is reached ?

(A) 0.66 A
(B) 0.29A
(C) 0 A
(D) 0.14 A

Ans : (C)

Hint : 

Equivalent emf of loop
(2 – 2) V = 0
∴ I = 0

$2 - 8I - 8I - 2 = 0$

$-8I = 0$

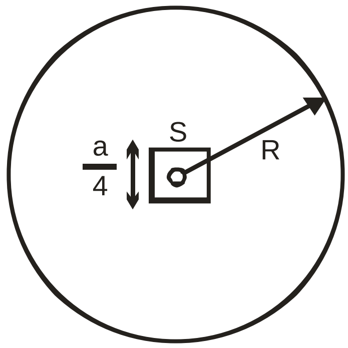
$I = 0$

28. Consider a conducting wire of length L bent in the form of a circle of radius R and another conductor of length ‘a’ (a <<R) is bent in the form of a square. The two loops are then placed in same plane such that the square loop is exactly at the centre of the circular loop. What will be the mutual inductance between the two loops ?

(A) $\mu_0 \frac{\pi a^2}{L}$
(B) $\mu_0 \frac{\pi a^2}{16L}$
(C) $\mu_0 \frac{\pi a^2}{4L}$
(D) $\mu_0 \frac{a^2}{4\pi L}$

Ans : (B)

Hint :



$2\pi R = L$ ....(1)

4s = a .... (2)

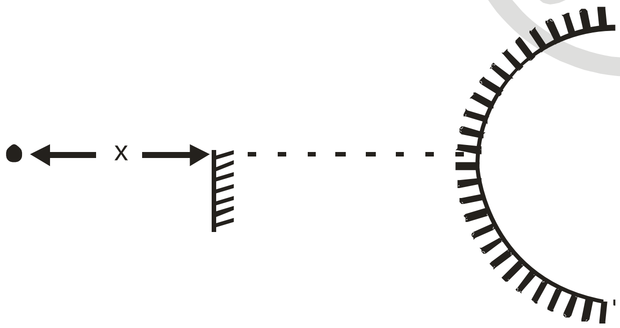
$\frac{\mu_0 I \times S^2}{2R} = \phi$, $M = \frac{\mu_0 S^2}{2R}$

$= \frac{\mu_0}{2} \times \frac{a^2 \times 2\pi}{16 \times L} = \frac{\mu_0 a^2 \pi}{16L}$

29. An object is placed 60 cm in front of a convex mirror of focal length 30 cm. A plane mirror is now placed facing the object in between the object and the convex mirror such that it covers lower half of the convex mirror. What should be the distance of the plane mirror from the object so that there will be no parallax between the images formed by the two mirrors ?

(A) 40 cm (B) 30 cm (C) 20 cm (D) 15 cm

Ans : (A)



Hint : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{60} = \frac{1}{30}$

$v = \frac{60}{3} = \pm 20$ cm

$\therefore$ Distance between object and image = 60 + 20 = 80 cm

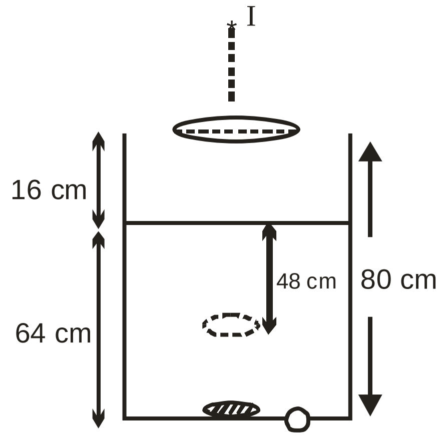
$\therefore$ x = 40 cm

30. A thin convex lens is placed just above an empty vessel of depth 80 cm. The image of a coin kept at the bottom of the vessel is thus formed 20 cm above the lens. If now water is poured in the vessel up to a height of 64 cm, what will be the approximate new position of the image. Assume that refractive index of water is 4/3.

(A) 21.33 cm above the lens
(B) 6.67 cm below the lens
(C) 33.67 cm above the lens
(D) 24 cm above the lens

Ans : (A)

Hint : $u = -80$ cm $v = +20$ cm



$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} = \frac{1}{20}+\frac{1}{80}=\frac{5}{80}$$

$f = 16$ cm

$$u' = 16+\frac{64\times 3}{4} = 64 \text{ cm}$$

$$\frac{1}{16}=\frac{1}{v}+\frac{1}{64} \quad \therefore \frac{1}{v}=\frac{1}{16}-\frac{1}{64}=\frac{3}{64}$$

$v = 21.33$ above the lens

Category II (Q31 to Q 35)

Carry 2 marks each and only one option is correct. In case of incorrect answer or combination of more than one answer, ½ mark will be deducted

31. A conducting circular loop of resistance 20 Ω and cross-sectional area 20×10^{-2} m^2 is placed perpendicular to a spatially uniform magnetic field B, which varies with time t as $B=2\sin(50\pi t)$ T. Find the net charge flowing through the loop in 20 ms starting from t = 0

(A) 0.5 C
(B) 0.2 C
(C) 0 C
(D) 0.14 C

Ans : (C)

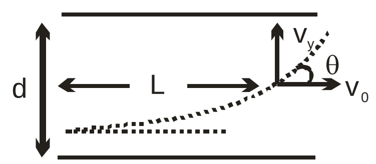
Hint : $\Delta q=\frac{\Delta\varphi}{R}=\frac{\varphi_2-\varphi_1}{R}=0$, $\Delta q = 0$ [$\varphi_1=\varphi_2$ = 0 at t=0 and t=20 ms]

32. A pair of parallel metal plates are kept with a separation 'd'. One plate is at a potential +V and the other is at ground potential. A narrow beam of electrons enters the space between the plates with a velocity v_0 and in a direction parallel to the plates. What will be the angle of the beam with the plates after it travels an axial distance L?

(A) $\tan^{-1}\left(\frac{eVL}{mdv_0}\right)$
(B) $\tan^{-1}\left(\frac{eVL}{mdv_0^2}\right)$
(C) $\sin^{-1}\left(\frac{eVL}{mdv_0}\right)$
(D) $\cos^{-1}\left(\frac{eVL}{mdv_0^2}\right)$

Ans : (B)

Hint : $t=\frac{L}{v_0}$



$$v_y=\frac{e}{m}.\frac{v}{d}\times\frac{L}{v_0}=\frac{evL}{mdv_0}$$

$\tan\theta = \frac{v_y}{v_x} = \frac{evL}{mdv_0.v_0}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{evL}{mdv_0^2}\right)$

33. A metallic block of mass 20 kg is dragged with a uniform velocity of 0.5 ms^{-1} on a horizontal table for 2.1s. The co-efficient of static friction between the block and the table is 0.10. What will be the maximum possible rise in temperature of the metal block if the specific heat of the block is 0.1 C.G.S unit ? Assume g=10 ms^{-2} uniform rise in temperature throughout the whole block. [Ignore absorption of heat by the table]

(A) 0.0025 ºC (B) 0.025 ºC (C) 0.001 ºC (D) 0.05 ºC

Ans : (B)

Hint : friction f = μ mg = 0.1×20×10 = 20

W_f = heat

$\Rightarrow$ (f.u)t = mc ΔT

$\Rightarrow 20\times0.5\times2.1 = 20\times0.1\times4.2\times10^3 . \Delta T$

$\Rightarrow \Delta T = 0.0025°C$

34. Consider an engine that absorbs 130 cal of heat from a hot reservoir and delivers 30 cal heat to a cold reservoir in each cycle. The engine also consumes 2J energy in each cycle to overcome friction. If the engine works at 90 cycles per minute, what will be the maximum power delivered to the load ? [Assume the thermal equivalent of heat is 4.2J/cal]

(A) 816 W (B) 819 W (C) 627 W (D) 630 W

Ans : (C)

Hint : work done per cycle = 100×4.2–2 = 418J

Total work done = 418 × 90

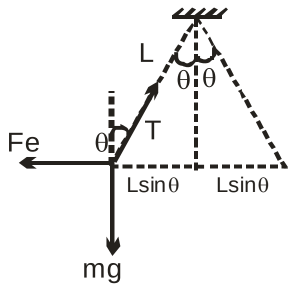
Power = $\frac{418\times90}{60}$ = 627 W

35. Two pith balls, each carrying charge +q are hung from a hook by two strings. It is found that when each charge is tripled, angle between the strings double. What was the initial angle between the strings ?

(A) 30° (B) 60° (C) 45° (D) 90°

Ans : (B)

Hint : T sinθ = Fe, T cosθ = mg



$\tan\theta = \frac{Fe}{mg} = \frac{k.q^2}{(2L\sin\theta)^2.mg}$

In 2nd case

$\tan 2\theta = \frac{kq\,q^2}{(2L\sin 2\theta)^2.mg}$

$\frac{\tan 2\theta}{\tan\theta} = \frac{9\sin^2\theta}{\sin^2 2\theta}$

$\frac{2}{1-\tan^2\theta} = \frac{9}{4\cos^2\theta} = \frac{9}{4}\sec^2\theta$

$\frac{2}{1-\tan^2\theta} = \frac{9}{4}(1-\tan^2\theta)$

$\tan^2\theta = x, \quad \frac{2}{1-x} = \frac{9}{4}(1+x)$

$\Rightarrow 8 = 9 - 9x^2 \Rightarrow x = \frac{1}{3} \Rightarrow \tan^2\theta = \frac{1}{3} \Rightarrow \theta = 30°$

Angle between the strings = $2\theta = 60°$

Category III (Q36 to Q40)

Carry 2 marks each and one or more option (s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero mark will be awarded.

36. A point source of light is used in an experiment of photo-electric effects. If the distance between the source and the photo-electric surface is doubled, which of the following may result ?

(A) Stopping potential will be halved

(B) photo-electric current will decrease

(C) Maximum kinetic energy if photo-electrons will decrease

(D) Stopping potential will increase slightly

Ans : (B)

Hint : Intensity $I \propto \frac{1}{d^2}$

and photo-electric current $\propto$ intensity

$\therefore$ Current will decrease

37. Two metallic spheres of equal outer radii are found to have same moment of inertia about their respective diameters. Then which of the following statement(s) is/are true ?

(A) The two spheres have equal masses

(B) The ratio of their masses is nearly 1.67 : 1

(C) The spheres are made of different materials

(D) Their rotational kinetic energies will be equal when rotated with equal uniform angular speed about their respective diameters

Ans : (D)

Hint : Inner radius is not given

$\therefore$ only option (D) is correct

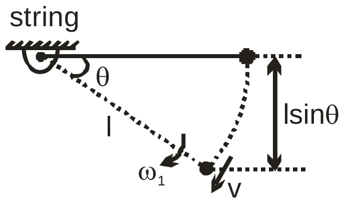
38. A simple pendulum of length ℓ is displaced so that its taught string is horizontal and then released. A uniform bar pivoted at one end is simultaneously released from its horizontal position. If their motions are synchronous, what is the length of the bar ?

(A) $\frac{3\ell}{2}$ (B) ℓ (C) 2ℓ (D) $\frac{2\ell}{3}$

Ans : (A)

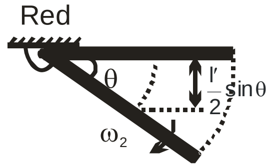
Hint : $v = \sqrt{2gh}$

$= \sqrt{2g.l\sin\theta}$



$$\omega_1 = \frac{v}{l} = \sqrt{\frac{2g\sin\theta}{l}}$$

$$mgh = \frac{1}{2}I\omega^2$$



$$\Rightarrow mg.\frac{l'\sin\theta}{2} = \frac{1}{2}\times\frac{ml^2}{3}\times\omega_2^2$$

$$\omega_2 = \sqrt{\frac{3g\sin\theta}{l'}} = \omega_1 \quad \Rightarrow \frac{2}{l} = \frac{3}{l'} \quad \Rightarrow \boxed{l' = \frac{3}{2}l}$$

39. A 400Ω resistor, a 250 mH inductor and a 2.5 μF capacitor are connected in series with an AC source of peak voltage 5V and angular frequency 2kHz. What is the peak value of the electrostatic energy of the capacitor ?

(A) 2 μJ (B) 2.5 μJ (C) 3.33 μJ (D) 5 μJ

Ans : (D)

Hint : The angular frequency is 2KHz. The unit given is incorrect Assuming it to be in radian / sec

$X_L = 2\times10^3\times250\times10^{-3} = 500\Omega$

$$X_C = \frac{1}{2.5\times10^{-6}\times2\times10^3} = 200\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 500\Omega$$

$$(V_C)_{peak} = i_{peak}X_C = \frac{(V_S)_{peak}}{Z}X_C = \frac{5}{500}\times200 = 2V$$

$$(U_C)_{max} = \frac{1}{2}\times2.5\times10^{-6}\times(2)^2 = 5\mu J$$

40. A charged particle moves with constant velocity in a region where no effect of gravity is felt but an electrostatic field $\vec{E}$ together with a magnetic field $\vec{B}$ may be present. Then which of the following cases are possible ?

(A) $\vec{E}\neq0, \vec{B}\neq0$ (B) $\vec{E}\neq0, \vec{B}=0$ (C) $\vec{E}=0, \vec{B}=0$ (D) $\vec{E}=0, \vec{B}\neq0$

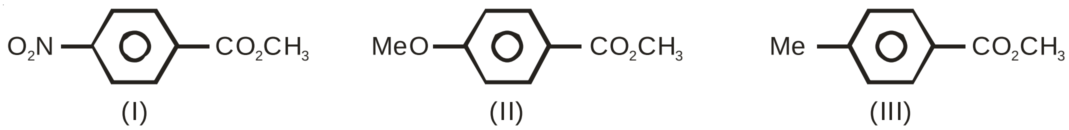
Ans : (A,C,D)

Hint :

CHEMISTRY

CATEGORY - I (Q41 to Q70)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

41. 

For the above three esters, the order of rates of alkaline hydrolysis is

(A) I > II > III (B) II > III > I (C) I > III > II (D) III > I > II

Ans : (C)

Hint : Alkaline hydrolysis is a nucleophilic substitution reaction on ester. Hence, presence of electron withdrawing group promotes nucleophilic attack

– NO_2 – R and – I

– OMe + R, – I

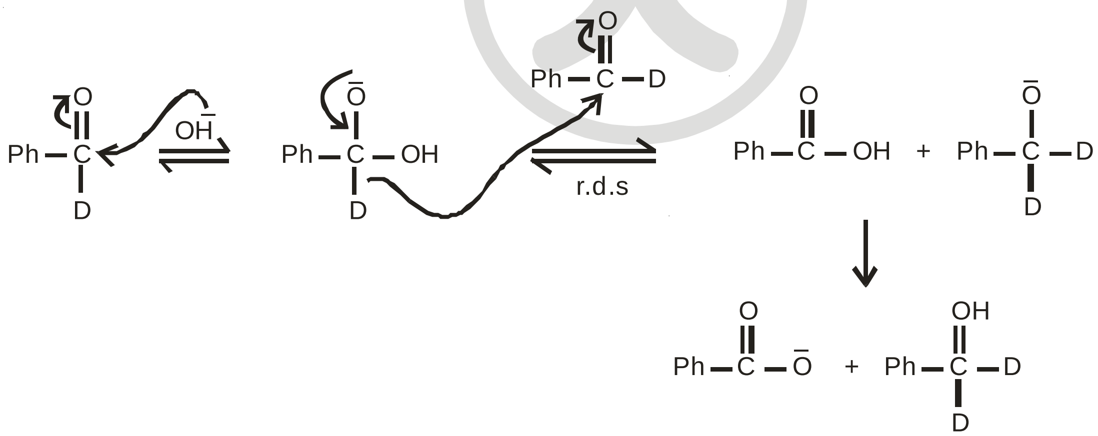
– Me + I and Hyperconjugation

42. Ph – CDO $\xrightarrow[\text{Warm}]{\text{50\% aq. NaOH}}$ Ph – $COO^{\ominus}H$ + an alcohol. This alcohol is

(A) PH – CHD – OH (B) Ph – CHD – OD (C) Ph – CD_2 – OH (D) Ph – CD_2 – OD

Ans : (C)

Hint : Hydride shift, here D^-, shift decides the product



43. The correct order of acidity for the following compounds is :

(A) II < IV < III < I (B) II < III < I < IV (C) II < III < IV < I (D) III < II < I < IV

Ans : (B)

Hint : Compound IV is a carboxylic acid whereas others are alcohols. So IV is most acidic, amongst the alcohols. Presence of $-NO_2$ increases acidic strength while the presence of $-CH_3$ lowers. Amongst I and III, more acidic is I as $-NO_2$ exerts it's –R influence in para position.

44. For the following carbocations the correct order of stability is

I : $\overset{\oplus}{C}H_2 - COCH_3$ II : $\overset{\oplus}{C}H_2 - OCH_3$ III : $\overset{\oplus}{C}H_2 - CH_3$

(A) III < II < I (B) II < I < III (C) I < II < III (D) I < III < II

Ans : (D)

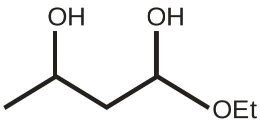
Hint : $\overset{+}{C}H_2 - \ddot{O}CH_3 \longleftrightarrow CH_2 = \overset{+}{O}CH_3$

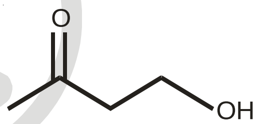
All atoms is octet
(most stable amongst these three)

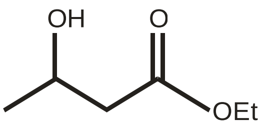
$\overset{+}{C}H_2 - \overset{O}{\overset{\|}{C}} - CH_3$ presence of electron withdrawing group destabilises it. (least stable)

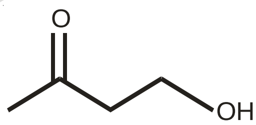
$\overset{+}{C}H_2 - CH_3$ Hyperconjugation of $-CH_3$ stabilises the ion to some extent but not as effectively as $-OCH_3$

45. The reduction product of ethyl 3-oxobutanoate by $NaBH_4$ in methanol is

(A) 

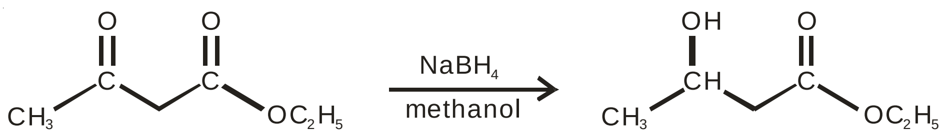
(B) 

(C) 

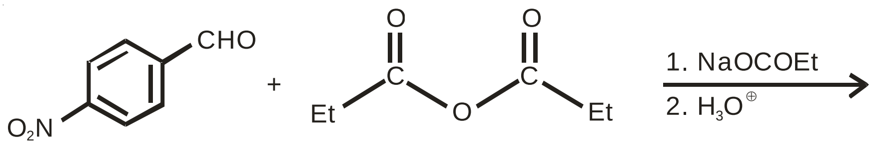
(D) 

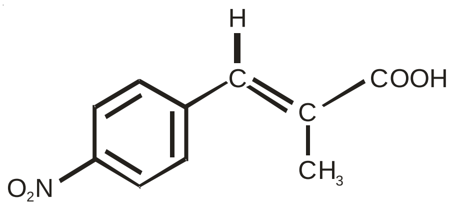
Ans : (C)

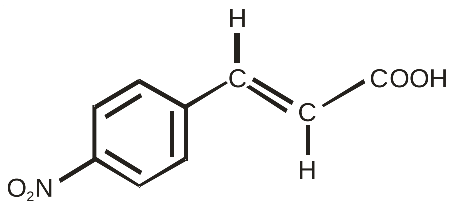
Hint : $NaBH_4$ reduces Ketone but not ester

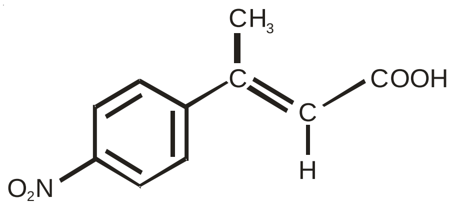


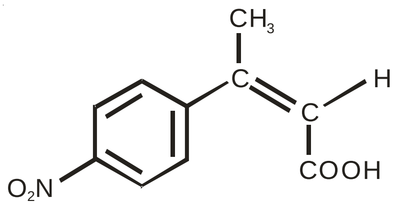
46. What is the major product of the following reaction ?



(A) 

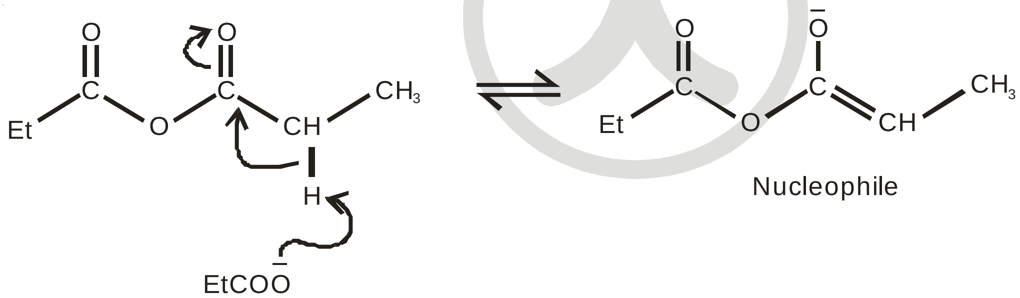
(B) 

(C) 

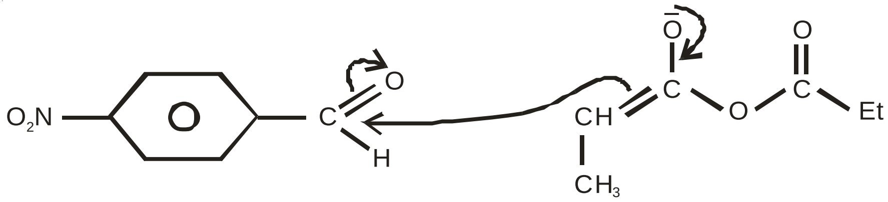
(D) 

Ans : (A)

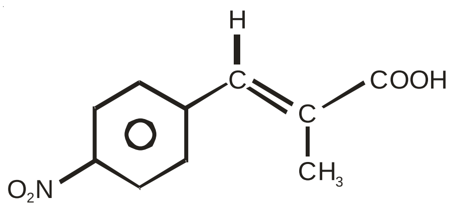
Hint : It's perkin reaction. The reaction starts with generation of nucleophile from propanoic anhydride as follows :



Then onwards, the nucleophile attacks the –CHO group as shown :



Finally, it goes on to give :



47. The maximum number of electrons in an atom in which the last electron filled has the quantum numbers n = 3, l = 2 and m = –1 is

(A) 17 (B) 27 (C) 28 (D) 30

Ans : (D)

Hint : n = 3 and l = 2 means 3d-orbital. Since five orientations of d-orbitals are degenerate, m = –1 can be assigned to any one. For maximum number of electrons as asked, answer should be atomic number 30.

48. In the face-centred cubic lattice structure of gold the closest distance between gold atoms is ('a' being the edge length of the cubic unit cell)

(A) $a\sqrt{2}$ (B) $\frac{a}{\sqrt{2}}$ (C) $\frac{a}{2\sqrt{2}}$ (D) $2\sqrt{2}a$

Ans : (B)

Hint : Closest distance between two gold atoms in fcc lattice of gold is $\frac{a}{\sqrt{2}}$. This is the distance between the corner atom and the closest face centre atom.

49. The equilibrium constant for the following reactions are given at 25°C

$2A \rightleftharpoons B + C, K_1 = 1.0$

$2B \rightleftharpoons C + D, K_2 = 16$

$2C + D \rightleftharpoons 2P, K_3 = 25$

The equilibrium constant for the reaction $P \rightleftharpoons A + \frac{1}{2}B$ at 25°C is

(A) $\frac{1}{20}$ (B) 20 (C) $\frac{1}{42}$ (D) 21

Ans : (A)

Hint : We can manipulate the given equations as follows

$\frac{1}{2}B + \frac{1}{2}C \rightleftharpoons A,$ $K_1' = 1$

$\frac{1}{2}C + \frac{1}{2}D \rightleftharpoons B,$ $K_2' = \frac{1}{\sqrt{16}}$

$P \rightleftharpoons C + \frac{1}{2}D,$ $K_3' = \frac{1}{\sqrt{25}}$

$P \rightleftharpoons A + \frac{1}{2}B,$ $K_{final} = 1 \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

50. Among the following, the ion which will be more effective for flocculation of $Fe(OH)_3$ sol. is

(A) PO_4^{3-} (B) SO_4^{2-} (C) SO_3^{2-} (D) NO_3^-

Ans : (A)

Hint : $Fe(OH)_3$ is a positive sol. PO_4^{3-} carrying the highest –ve charge amongst the given ions is most effective

51. The mole fraction of ethanol in water is 0.08. Its molality is

(A) 6.32 mol kg^{-1} (B) 4.83 mol kg^{-1} (C) 3.82 mol kg^{-1} (D) 2.84 mol kg^{-1}

Ans : (B)

Hint : Molality $= \frac{x_{EtOH}}{x_{H_2O}} \times \frac{1000}{MW_{solvent}}$

$= \frac{0.08}{0.92} \times \frac{1000}{18}$

$= 4.83 \text{ mol kg}^{-1}$

52. 5 ml of 0.1 M $Pb(NO_3)_2$ is mixed with 10 ml of 0.02 M KI. The amount of PbI_2 precipitated will be about

(A) 10^{-2} mol (B) 10^{-4} mol (C) 2×10^{-4} mol (D) 10^{-3} mol

Ans : (B)

	$Pb(NO_3)_2\,_{(aq.)}$ +	$2KI_{(aq.)}$ ⟶	$PbI_2 \downarrow$ +	$2KNO_3\,_{(aq.)}$
Hint : Initial milimole	5×0.1 $= 0.5$	10×0.02 $= 0.2$	0	0
Final milimole	0.1	0	0.1	0.2

mole of PbI_2 PPt $= \frac{0.1}{1000} = 1\times 10^{-4}$

53. At 273 K temperature and 76 cm Hg pressure, the density of a gas is $1.964 gL^{-1}$. The gas is

(A) CH_4 (B) CO (C) He (D) CO_2

Ans : (D)

Hint : Molar mass of gas $(M) = \frac{dRT}{P} = \frac{1.964\times 0.0821\times 273}{1}$

= 44g

Hence, the gas is CO_2

54. Equal masses of ethane and hydrogen are mixed in a empty container at 298 K. The fraction of total pressure exerted by hydrogen is

(A) 15 : 16 (B) 1 : 1 (C) 1 : 4 (D) 1 : 6

Ans : (A)

Hint : Let mass of ethane and H_2 are x g.

$n_{H_2} = \frac{x}{2}$ and $n_{C_2H_6} = \frac{x}{30}$

$$\frac{P_{H_2}}{P_T} = \frac{n_{H_2}}{n_{H_2} + n_{C_2H_6}} = \frac{\frac{x}{2}}{\frac{x}{2} + \frac{x}{30}} = \frac{15}{16}$$

55. An ideal gas expands adiabatically against vaccum. Which of the following is correct for the given process?

(A) $\Delta S = 0$ (B) $\Delta T = -ve$ (C) $\Delta U = 0$ (D) $\Delta P = 0$

Ans : (C)

Hint : For adiabatic free expansion of an ideal gas

$dq = 0$

$dU = 0$

$dH = 0$

$dT = 0$

56. K_f (water) = 1.86 K kg mol^{-1}. The temperature at which ice begins to separate from a mixture of 10 mass % ethylene glycol is

(A) – 1.86 °C (B) – 3.72 °C (C) – 3.3 °C (D) –3 °C

Ans : (C)

Hint : $\Delta T_f = K_f \times m$

$$=1.86\left(\frac{10}{62}\times\frac{1000}{90}\right)$$

= 3.3°C

∴ Temparature at which ice begins to separate = 3.3°C

57. The radius of the first Bohr orbit of a hydrogen atom is 0.53×10^{-8} cm. The velocity of the electron in the first Bohr orbit is

(A) 2.188×10^8 cm s^{-1} (B) 4.376×10^8 cm s^{-1} (C) 1.094×10^8 cm s^{-1} (D) 2.188×10^9 cm s^{-1}

Ans : (A)

Hint : r = 0.53×10^{-8} cm = $0.53\times 10^{-8}n^2$

∴ n = 1

V = 2.18×10^8 cm/s

58. Which of the following statements is not true for the reaction $2F_2 + 2H_2O \rightarrow 4HF + O_2$?

(A) F_2 is more strongly oxidising than O_2

(B) F – F bond is weaker than O = O bond

(C) H – F bond is stronger than H – O bond

(D) F is less electronegative than O

Ans : (D)

Hint : F is more electronegative than O

59. The number of unpaired electrons in the uranium ($_{92}U$) atom is :

(A) 4 (B) 6 (C) 3 (D) 1

Ans : (A)

Hint : $_{92}U$: $[R_n]^{86}\ 5f^3 6d^1 7s^2$

60. How and why does the density of liquid water change on prolonged electrolysis ?

(A) Decreases, as the proportion of H_2O increases

(B) Remains unchanged

(C) Increases, as the proportion of D_2O increases

(D) Increases, as the volume decreases

Ans : (C)

Hint : H_2O decomposes preferentially. Consequently D_2O concentration increases and density increases.

61. The difference between orbital angular momentum of an electron in a 4f orbital and another electron in a 4s orbital is

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

Ans : (A)

Hint : Orbital angular momentum $= \sqrt{\ell(\ell+1)}\frac{h}{2\pi}$

For 4f orbital; $\ell = 3$ $\therefore$ Orbital angular momentum $= \sqrt{3(3+1)}\dfrac{h}{2\pi} = 2\sqrt{3}\dfrac{h}{2\pi}$

For 4s orbital; $\ell = 0$ $\therefore$ Orbital angular momentum = 0

$\therefore$ Difference $= \left(2\sqrt{3}\right)\dfrac{h}{2\pi}$

62. Which of the following has the largest number of atoms ?

(A) 1 g of Ag (B) 1 g of Fe (C) 1 g of Cl_2 (D) 1 g of Mg

Ans : (D)

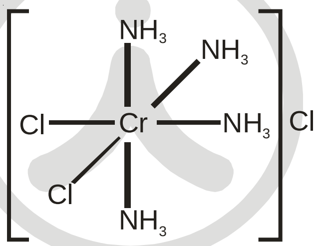
Hint : Number of atoms of $Ag = \dfrac{1}{108} \times N_A = 0.009\,N_A$

Number of atoms of $Fe = \dfrac{1}{56} \times N_A = 0.0178 N_A$

Number of atoms of $Cl_2 = \dfrac{1}{71} \times 2 \times N_A = 0.028 N_A$

Number of atoms of $Mg = \dfrac{1}{24} N_A = 0.041 N_A$ [largest]

63. Indicate the correct IUPAC name of the co-ordination compound shown in the figure.

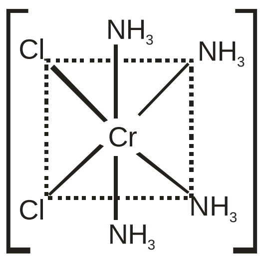


(A) Cis-dichlorotetraminochromium (III) chloride (B) Trans-dichlorotetraminochromium (III) chloride

(C) Trans-tetraminedichlorochromium (III) chloride (D) Cis-tetraamminedichlorochromium (III) chloride

Ans : (D)

Hint :



Cis-tetraamminedichlorochromium (III) chloride

64. What will be the mass of one atom of ^{12}C ?

(A) 1 a.m.u. (B) 1.9923×10^{-23}g (C) 1.6603×10^{-22}g (D) 6 a.m.u.

Ans : (B)

Hint : Mass of 1 atom of ^{12}C = 12 a.m.u. = $12 \times 1.66 \times 10^{-24}$g = 1.9923×10^{-23}g

65. Bond order of He_2, He_2^+ and He_2^{2+} are respectively :

(A) $1, \frac{1}{2}, 0$ (B) $0, \frac{1}{2}, 1$ (C) $\frac{1}{2}, 1, 0$ (D) $1, 0, \frac{1}{2}$

Ans : (B)

Hint : $He_2\left(4e^-\right) = \sigma(1s)^2, \sigma^*(1s)^2$; $B.O = \frac{2-2}{2} = 0$

$He_2^+\left(3e^-\right) = \sigma(1s)^2 \sigma^*(1s)^1$; $B.O = \frac{2-1}{2} = \frac{1}{2}$

$He_2^{+2}\left(2e^-\right) = \sigma(1s)^2$; $B.O. = \frac{2-0}{2} = 1$

66. To a solution of a colourless efflorescent sodium salt, when dilute acid is added, a colourless gas is evolved along with formation of a white precipitate. Acidified dichromate solution turns green when the colourless gas is passed through it. The sodium salt is

(A) Na_2SO_3 (B) Na_2S (C) $Na_2S_2O_3$ (D) $Na_2S_4O_6$

Ans : (C)

Hint : $\underset{\text{(colourless,efflorescent)}}{Na_2S_2O_3(s)} + 2H^+(\text{dil.}) \rightarrow \underset{\text{(white ppt)}}{S(s)} + 2Na^+(aq.) + SO_2\uparrow + H_2O(\ell)$

$K_2Cr_2O_7(aq.) + 3SO_2(g) + 2H^+(aq.) \xrightarrow{\rightarrow} 2K^+(aq.) + \underset{\text{(green)}}{2Cr^{+3}(aq)} + H_2O(\ell) + 3SO_4^{-2}(aq)$

67. The reaction for obtaining the metal (M) from its oxide (M_2O_3) ore is given by

$M_2O_3(s) + 2Al(\ell) \xrightarrow{\text{Heat}} Al_2O_3(\ell) + 2M(s), (s = \text{solid}, \ell = \text{liquid})$

in that case, M is

(A) Copper (B) Calcium (C) Iron (D) Zinc

Ans : (C)

Hint : Thermite Process

68. In the extraction of Ca by electro reduction of molten $CaCl_2$ some CaF_2 is added to the electrolyte for the following reason :

(A) To keep the electrolyte in liquid state at temperature lower than the m. p. of $CaCl_2$

(B) To effect precipitation of Ca

(C) To effect the electrolysis at lower voltage

(D) To increase the current efficiency

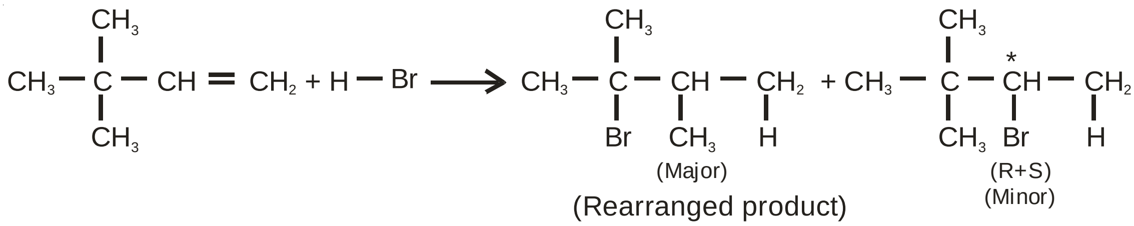
Ans : (A)

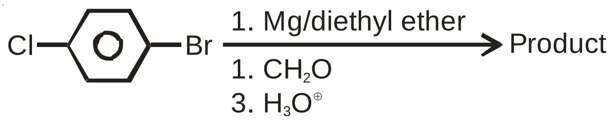
Hint : CaF_2 decreases the melting point of $CaCl_2$

69. The total number of alkyl bromides (including stereoismers) formed in the reaction $Me_3C - CH = CH_2 + HBr \rightarrow$ will be

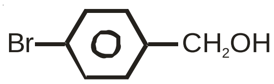
(A) 1 (B) 2 (C) 3 (D) No bromide forms

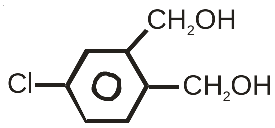
Ans : (C)

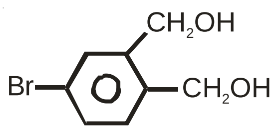
Hint : 

70. 

This product in the above reaction is

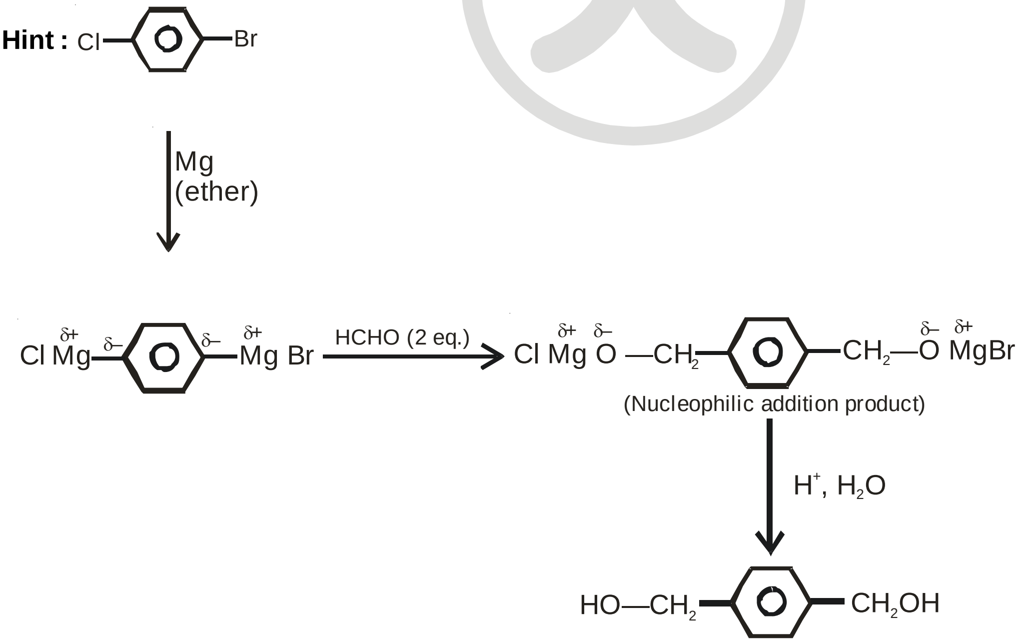
(A) 

(B) 

(C) 

(D) 

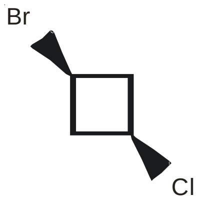
Ans : (D)

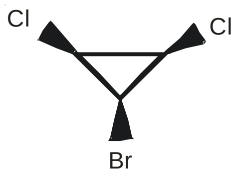
Hint : 

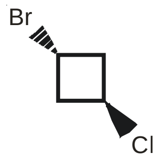
CATEGORY - II (Q71 to Q75)

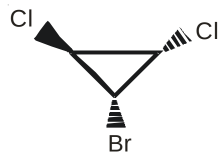
Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

71. Which of the following compounds is asymmetric?

(A) 

(B) 

(C) 

(D) 

Ans : (D)

Hint : No plane of symmetry as well as centre of symmetry is present,

72. For a reaction 2A + B → P, when concentration of B alone is doubled, $t_{\frac{1}{2}}$ does not change and when concentrations of both A and B is doubled, rate increases by a factor of 4. The unit of rate constant is,

(A) s^{-1} (B) L $mol^{-1}s^{-1}$ (C) mol $L^{-1}s^{-1}$ (D) L^2 $mol^{-2}s^{-1}$

Ans : (B)

Hint :

Rate law: $r = k[A]^{\alpha}[B]^{\beta}$ B follows first order kinetics. ∴ $\beta = 1$

$$\frac{r_2}{r_1} = \frac{[2A]^{\alpha}[2B]^{\beta}}{[A]^{\alpha}[B]^{\beta}}$$

or, $4 = 2^{\alpha+\beta}$

or, $\alpha + \beta = 2$

or, $\alpha = 1$

∴ overall order of reaction = 2

Hence unit of rate constant = L.mol^{-1}.S^{-1}

73. A solution is saturated with $SrCO_3$ and SrF_2. The $[CO_3^{2-}]$ is found to be 1.2×10^{-3}M. The concentration of F– in the solution would be

(A) 3.7×10^{-6} M (B) 3.2×10^{-3} M (C) 5.1×10^{-7} M (D) 3.7×10^{-2} M

Given : K_{sp} ($SrCO_3$) = 7.0×10^{-10}, K_{sp} (SrF_2) = 7.9×10^{-10}

Ans : (D)

Hint : $SrCO_3(s) \rightleftarrows Sr^{2+}(aq) + CO_3^{2-}(aq)$

$S_1 + S_2$ S_1

$SrF_2(s) \rightleftarrows Sr^{2+}(aq) + 2F^{-}(aq)$

$S_1 + S_2$ $2S_2$

Where S_1 and S_2 are solubilities of $SrCO_3$(s) and SrF_2 (s) respectively.

$S_1 = 1.2 \times 10^{-3}$M (Given)

$K_{sp}(SrCO_3) = [Sr^{2+}][CO_3^{-2}]$

$7 \times 10^{-10} = (S_1 + S_2)\,S_1$ -------- (i)

$K_{sp}(SrF_2) = [Sr^{2+}][F^-]^2$

$7.9 \times 10^{-10} = (S_1 + S_2)(2S_2)^2$ -------- (ii)

Dividing equation (ii) by equation (i)

$$\frac{7.9 \times 10^{-10}}{7 \times 10^{-10}} = \frac{4S_2^{\,2}}{S_1}$$

$$\therefore\ S_2 = \sqrt{\frac{7.9}{7} \times \frac{1.2 \times 10^{-3}}{4}}$$

$= 1.84 \times 10^{-2}$

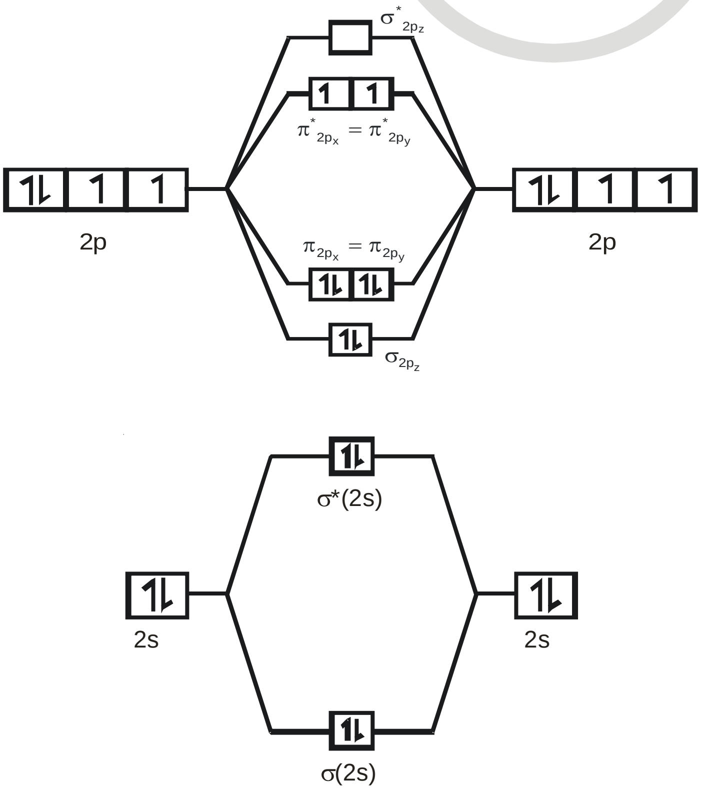
$\therefore\ [F^-] = 2S_2 = 2 \times 1.84 \times 10^{-2} = 3.68 \times 10^{-2}$ M

74. A homonuclear diatomic gas molecule shows 2-electron magnetic moment. The one-electron and two-electron reduced species obtained from above gas molecule can act as both oxidizing and reducing agents. When the gas molecule is one-electron oxidized the bond length decreases compared to the neutral molecule. The gas molecule is

(A) N_2 (B) Cl_2 (C) O_2 (D) B_2

Ans : (C)

Hint :



Molecular orbital diagram of O_2

75. $CH_3 — O — CH_2 — Cl \xrightarrow[\Delta]{aq.^{(-)}OH} CH_3 — O — CH_2 — OH$

Which information below regarding this reaction is applicable?

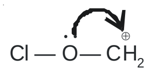
(A) It follows S_N2 pathway, because it is a primary alkyl chloride.

(B) It follows S_N1 pathway, because the intermediate carbocation is resonance stabilized.

(C) S_N1 pathway is not followed, because the intermediate carbocation is destabilised by –1 effect of oxygen

(D) A mixed S_N1 and S_N2 pathway is followed.

Ans : (D)

Hint : 

Stabilised by resonance $\Rightarrow S_N1$

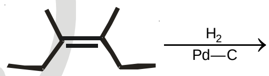
S_N2 is also facilitated by $\overset{\ominus}{O}H$ due to resonance stablization of transition state.

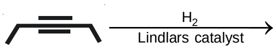
CATEGORY - III (Q.76 to Q.80)

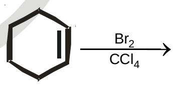
Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero mark will be awarded.

76. Which of the following reactions give(s) a *meso*-compound as the main product?

(A) 

(B) 

(C) 

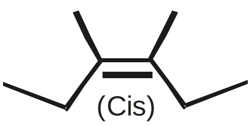
(D) 

Ans : (A)

Hint : 

(Trans) $\xrightarrow[\text{in } CCl_4]{Br_2}$ anti addition of bromine

↓



$\xrightarrow[Pd—C]{H_2}$ Meso compound (Catalytic hydrogenation gives syn addition)

77. For spontaneous polymerization, which of the following is (are) correct?

(A) Δ G is negative (B) Δ H is negative (C) Δ S is positive (D) Δ S is negative

Ans : (A,B, D)

Hint : ΔG is negative as process is spontaneous.

ΔS is negative due to association.

Therefore to make ΔG negative, ΔH must be negative.

78. Which of the following statement(s) is/are incorrect:

(A) A sink of SO_2 pollutant is O_3 in the atmosphere.

(B) FGD is a process of removing NO_2 from atmosphere

(C) NO_x in fuel gases can be removed by alkaline scrubbing

(D) The catalyst used to convert CCl_4 to CF_4 by HF is SbF_5

Ans : (A, B and D)

Hint : Oxides of nitrogen can be removed by alkaline scrubbing.

79. SiO_2 is attacked by which one / ones of the following?

(A) HF (B) conc. HCl (C) hot NaOH (D) Fluorine

Ans : (A, C, D)

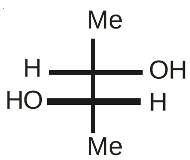
Hint : $SiO_2\,(s) + F_2\,(g) \rightarrow SiF_4\,(aq.) + O_2\,(g)$

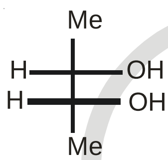
$SiO_2\,(s) + 6HF\,(aq.) \rightarrow H_2SiF_6\,(aq.) + 2H_2O\,(\ell)$

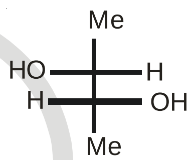
$SiO_2(s) + 2NaOH(aq.) \rightarrow Na_2SiO_3(s) + H_2O(\ell)$

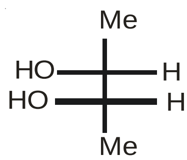
80. $Me - C \equiv C - Me \xrightarrow[EtOH,\ -33^\circ C]{Na/NH_3\,(liq.)} \underset{=}{X} \xrightarrow{dil.\ alkaline\ KMnO_4} Product(s).$

The product (s) from the above reaction will be

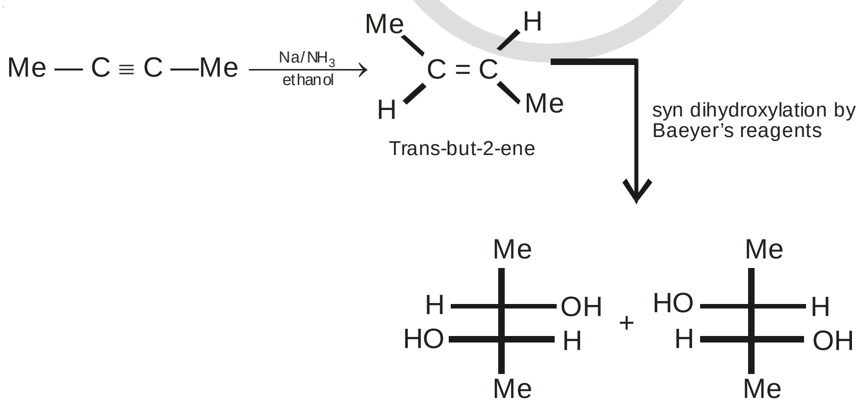
(A) 

(B) 

(C) 

(D) 

Ans : (A and C)

Hint : 

(Racemic mixture)

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